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Knowledge and Representations: The Meaning Making Process in the Curriculum of Mathematics

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CERTIFICATE OF ORIGINALITY

I hereby declare that this thesis is my own work and that, to the best of my knowledge and belief, it reproduces no material previously published or written, nor material that has been accepted for the award of any other degree or diploma, except where due acknowledgement has been made in the text.

Xia Li

Abstract

The purpose of this study is to understand the representations of mathematical knowledge in a series of co-related pedagogic discourses in the curriculum of mathematics in Hong Kong. Pythagoras' Theorem was selected as the focus of this study. Co-related pedagogic discourses such as the mathematics syllabus (EDB, 1999), the mathematics curriculum guideline (HKEAA, 2007), mathematics textbooks (e.g. Wong & Wong, 2009) and the mathematics examination paper (HKEAA, 2012), were selected as the source of data from which instances of the representation of Pythagoras' Theorem were examined and compared. As for the theoretical framework of this study, recontextualisation (Bernstein, 1990) has been reconciled with reinstantiation (Painter et al., 2012) in order to model the phenomenon of knowledge representation from a linguistic perspective. The findings of this study will contribute to the understanding of knowledge delocation and relocation in mathematics and other subjects from a linguistic perspective.

Chapter One introduces the need to understand mathematics, the changes of the secondary mathematics school curriculum in Hong Kong and the rationale for studying the relations between the curriculum documents. Chapter Two reviews the research in mathematical knowledge structures and the semiotic nature of mathematics. Chapter Three connects social semiotics to sociological perspective, offering the understanding of the relationship between different pedagogic discourses from a linguistic perspective. Chapter Four is concerned with the research methodology and provides the analytical models to understand research data. Chapter Five analyses research data, focusing on the multisemiotic phenomenon in the curriculum of mathematics. Chapter Six discusses the findings of this research, presenting the reasons for using systemic functional theory and sociological approaches to investigate knowledge structures in mathematics and other areas. Chapter Seven concludes this research and provides insights that will illuminate other studies interested in understanding the knowledge and representation of knowledge in school system.

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List of Abbreviations

EDB	Education Bureau of Hong Kong
HKEAA	Hong Kong Examinations and Assessment Authority
HKDSE	Hong Kong Diploma of Secondary Education
HKCEE	Hong Kong Certificate of Education Examination
HKALE	Hong Kong Advanced Level Examination
SFL	Systemic Functional Linguistics
SFMDA	Systemic Functional Multimodal Discourse Analysis
e.g.	exempli gratia, meaning "for example"
et al.	et alia, meaning "and others"
etc.	et cetera, meaning "and other similar things"
i.e.	id est, meaning "that is"

Chapter One – Introduction

1.1 Introduction

This initial chapter is composed of three crucial components. Section 1.2 introduces the important role of mathematics with the underlying principle that "Mathematics is the queen and servant of science" being reiterated. Section 1.3 describes how English is used for academic purposes in Hong Kong. Section 1.4 discusses the structure of the "Hong Kong Diploma of Secondary Education" (HKESE hereafter) before narrowing the focus to the crucial role played by mathematics in the Hong Kong secondary school education system. Section 1.5 offers a summary of the present study, outlining key points in subsequent chapters.

1.2 Mathematics is the queen and servant of science

> *Mathematics is the Queen of the Sciences and Arithmetic the Queen of Mathematics. She often condescends to render service to astronomy and other natural sciences, but under all circumstances the first place is her due.*

Above is the quotation by C.F. Gauss, the famous German Mathematician. This quotation was translated and quoted by Professor Eric Temple Bell, Emeritus Professor at California Institute of Technology in his book "Mathematics: Queen and Servant of Science" (1951, p. 1). Bell (1951, p. 2) argued that without mathematics, the revolution of modern physics would have never taken place. Following the arguments of Gauss and Bell, there is an internal relationship between mathematics and other disciplines. Metaphorically speaking, by saying that mathematics is both the queen and servant, it means that mathematics is the basis of other natural sciences such as astronomy, physical, chemistry, biology being invented and perhaps further developed. The internal relationship between mathematics and other disciplines is such that the theoretical foundation and calculation scheme of mathematics has the potential to be deployed into other scientific theories and subjects. For example, Sir Isaac Newton's "*Mathematical Principles of Natural Philosophy*" was established by deploying basic mathematical principles into the consolidation of his "Three Laws of Motion", appearing in the form of mathematical symbolic equations.

Speaking from a linguistic perspective, Halliday (2004, p. 217) suggests that the system of mathematics is "constructed to explain them (physical systems)" based on his work of classical Newtonian physics. For Halliday (2004), the systems of all natural sciences, such as biology, are constructed on a physical system. He argued that "a physical system ... is purely physical in nature; but a biological system is both biological and physical" (Halliday, 2004, p. 217). To extend Halliday's (2004) argumentation of physical and biological systems, and to also include Halliday's (2004) elaboration of mathematics, I will argue that a mathematical system is purely mathematical in nature; but a physical system is both mathematical and physical, confirming Bell's (1951) belief that "mathematics is the queen and servant of natural science". Therefore, as O'Halloran (2007a) suggests, "mathematics is none the less called 'the queen of the sciences' and is ranked first amongst the sciences" (p. 210).

Drawing from Halliday's (2005, Halliday and Matthiessen, 1999) ordered typology of system, mathematics is treated as a semiotic system. By saying mathematics is a semiotic system it is because, in social semiotic tradition, mathematics is a kind of "designed semiotics" (Halliday and Matthiessen, 2014, p. 20). Its ways of making meaning is designed (Halliday, 2002, p. 416) and "is grounded in the grammar of natural language" (Halliday, 2002, p. 392), such as the use of "grammatical metaphor, the union of nominalization and the recursive modification of the nominal group" (Halliday, 2004, p. 216). In other words, natural language is the metalanguage of mathematics because in the designing of mathematical theories, "the grammar of natural language" (Halliday, 2002, p. 392) is deployed and this grammar "enables mathematical expressions to be rendered in English, or Chinese, or other forms of distinctively human semiotic" (Halliday, 2003, p. 117). Following Halliday's (2002) elaboration, O'Halloran (2005) observes that "English is used as the metalanguage to teach mathematics" (p. 200). This relationship could also be observed when mathematics is written and taught in languages other than English.

In investigating the importance of mathematics from a linguistic perspective, Halliday (1978) states that "every language embodies some mathematical meanings in its semantic structure – ways of counting, measuring, classifying and so on" (p. 195).

In terms of the coverage of mathematics, it ranges from the basic calculating learned by toddlers before they enter kindergarten to the most advancing areas concerned by recipients of the Fields Medal.[1] The scope of the present study concentrates on secondary school mathematics in Hong Kong.

The reason for Hong Kong being selected is that English is used for academic purposes on the one hand, on the other hand, the Hong Kong mathematical curriculum is a standardized structure similar to other curriculum structures. For example, the aim of Cambridge International AS and A Level Mathematics, is to "balance knowledge, understanding and skills" (Cambridge International Examinations, 2014, p. 3), similar to the aim advocated by Educational Bureau of Hong Kong whose candidates are required to acquire "basic concepts, knowledge, properties and simple applications in real life situations" (EDB, 1999, p.6).

Different types of academic documents such as the mathematical syllabus (EDB, 1999), the curriculum guideline (HKEAA, 2007), the examination papers (i.e. HKEAA, 2012) include English versions. The next section will introduce the emerging phenomenon of English being used in educational contexts.

1.3 English as the medium of instruction in the mathematics curriculum

It is well documented that an increasing number of schools, curricula and subjects from primary to tertiary education, based on English as a medium of instruction (EMI hereafter), have emerged in places where English is not the first language (Evans, 2000). The use of English across the curriculum has been a great concern for both students and teachers who aim at successful and effective learning and teaching. The notion of EMI could extend to cover not only classroom data, such as the teachers' instruction and teacher–student interactions, but also the composition of written curriculum documents.

Several studies (e.g. Hoare, 2003 for Hong Kong; Liu, 2011 for Singapore) have discussed the use of English in secondary school curricula in Hong Kong and

[1] The Fields Medal is the most prestigious award in world mathematics.

Singapore where the majority of the population is Chinese. As Schleppegrell (2012) stated, "Learning to use language in ways that meet the school's expectations for advanced literacy task is a challenge for all students who have little opportunity for exposure to and use of such language outside of school" (p. viii). This challenge is twofold: one for the language and the other for the expected knowledge. For students in the EMI education system, "English is both a target and medium of education: they are not only learning the dominant language but they are learning in it and through it as well" (Gibbons, 2002, p. 258, & 2003, p. 247). For these students, "the construction of curriculum knowledge" as Gibbons (2002, p. 258) stated, "must go hand in hand with the development of the second language." Therefore, the understanding of curriculum documents that are in English requires not only the expected curriculum knowledge of the concerned subject (such as Mathematics in this study) but also competence in understanding English.

In response to the demand for both language and knowledge in Hong Kong, the newly launched HKDSE examination is considered appropriate "in terms of coverage, standard and wording" (HKEAA, 2011c, p. 15). "Coverage" and "standard" refer to the need for knowledge tested in the HKDSE examination being accurate and meeting the demands of the HKDSE education system, while "wording" refers to the language of the curriculum properly articulating the intended meaning.

The next section will introduce the background of the Hong Kong Diploma of Secondary Education" (HKDSE hereafter) and the important role played by mathematics as one compulsory subject.

1.4 HKDSE examination: general background

The year 2012 marked a significant transition in the examination system in Hong Kong's secondary schooling. Organized by the Hong Kong Examination and Assessment Authority (hereafter, HKEAA), the first HKDSE examination held "in late March to late May" (HKEAA, 2011a, p. 1) of 2012 by the HKEAA attracted "a total of 72,876 candidates"(HKEAA, 2011b, p. 1). This newly launched examination is Hong Kong's first attempt to "measure the attainment of

Secondary Six students who have completed a three-year senior secondary curriculum" (HKEAA, 2013, p. 1) in line with the implementation of the new academic system in the year 2009 by the Education Bureau of Hong Kong. Under this new academic system, all students in Hong Kong are required to complete six years of secondary education with three years for junior secondary school followed by three years for senior secondary school. By contrast, before 2009, the mainstream secondary schooling was composed of seven forms, "with exit points at Secondary 3, Secondary 5 and Secondary 7" (Adamson & Li, 2004, p. 53). Previously, two public examinations, the Hong Kong Certificate of Education Examination (hereafter, HKCEE) and the Hong Kong Advanced Level Examination (hereafter, HKALE), were designed for students attempting to leave secondary schooling at Secondary 5 and Secondary 7 respectively. Each had different purposes and functionalities. According to Choi (1999), "the HKCEE results are regarded as the basic qualification for employment" (p. 409), while after two more years of education, the HKALE examination was treated as the passport to tertiary education because "admission is primarily based on the applicant's HKALE results" (p. 409). Since 2012, the unified HKDSE examination replaced the previous two examinations as a single examination designed as suitable for all secondary school graduates. In response to this reformation of education policy, the education system together with different types of pedagogic discourses involved have all been substantially changed. This study investigates mathematics as an independent subject within the new HKDSE system. The approach will be a discourse analytical approach by focusing on different types of pedagogic discourses in their representation of mathematical knowledge. The substantial role of mathematics in the newly implemented HKDSE system is considered first.

1.4.1 Mathematics: a compulsory subject

In the HKDSE, there is one compulsory mathematics examination and two extended mathematics examinations (HKEAA, 2011). These comprise three independent subjects, namely compulsory HKDSE Mathematics, Calculus and Statistics, and Algebra and Calculus*.* Among these three subjects, only HKDSE Mathematics (HKEAA, 2012) is one of four compulsory core subjects to be completed by all secondary school graduates at the HKDSE level. As for the other two extended mathematics subjects, they are designed to "cater for students who intend to: pursue further studies which require more mathematics; or follow a career in fields such as natural sciences, computer sciences, technology or engineering" (HKEAA, 2007, p. 4). Compared to the two extended mathematics examinations, the compulsory core is oriented to meet the needs of all HKDSE candidates. According to the official registration statistics for the first HKDSE examination released on November 17, 2011 (HKEAA, 2011b), 55,796 candidates were registered for the compulsory Mathematics subject in their HKDSE examination. By way of contrast, 7,819 candidates registered for both the compulsory HKDSE Mathematics (HKEAA, 2012) and *Calculus and Statistics* subjects, while 8,376 candidates registered for both the compulsory HKDSE Mathematics (HKEAA, 2012) and *Algebra and Calculus*. Among the 72,876 candidates for the first HKDSE examination, 71,991 took the compulsory mathematics examination. The rest of the candidates quit HKDSE mathematics before the compulsory mathematics examination commenced.

1.4.2 The standard in composing the compulsory HKDSE Mathematics (HKEAA, 2012)

This new examination was designed to incorporate two previous public examinations – the Hong Kong Certificate of Education Examination (HKCEE) and the Hong Kong Advanced Level Examination (HKALE) – into a unified whole suitable for all Hong Kong Secondary school graduates. In order that the quality of the HKDSE examination is maintained, the process of "setting of examination papers" (HKEAA, 2011c) is overseen by "Moderation committee members" (HKEAA, 2011c, p. 15) to ensure that the design of examination papers follows a standard consistent with "the curriculum aims and assessment objectives" (HKEAA, 2011c, p. 15).

Based on this standard, the compulsory HKDSE Mathematics subject is the outcome of a critical consideration of the curriculum aims and assessment objectives. By stating that it is an outcome, I imply it is an intuitive perception that the compulsory HKDSE Mathematics is generated by the curriculum aims and assessment objectives. Since both the curriculum aims and assessment objectives were addressed in the mathematics syllabus (EDB, 1999) and mathematics curriculum guideline (HKEAA, 2007), it is fair to argue that the compulsory HKDSE Mathematics is influenced by both the syllabus (EDB, 1999) and curriculum guideline (HKEAA, 2007). The relationship between the compulsory HKDSE Mathematics and the two curriculum documents (EDB, 1999 and HKEAA, 2007) are examined in the present study. In addition to the syllabus (EDB, 1999), curriculum guideline (HKEAA, 2012) and the examination paper (HKEAA, 2012), as another important curriculum document the set textbook will also be considered.

1.5 Structure of the research

The purpose of this research is to discuss the meaning-making process in mathematics. Two distinctive but co-related streams, namely knowledge and representation, are investigated. Whether mathematical knowledge should be considered as the mathematical concept or whether knowledge should include both the concept and the representation, need also to be investigated as a high priority. Without devoting effort to differentiating these two distinctive ideas, the ensuing discussion will be unwieldy. Chapter 2 discusses and compares the current contributions in the field of mathematical knowledge structure. It also discusses the current research in the field of mathematical discourse, dealing with the meaning-making process in mathematics. At the end of Chapter 2, the scope of mathematical knowledge to be investigated in this study is presented. This scope outlined is the theoretical standpoint regarding what mathematical knowledge is treated in this present study. Chapter 3 introduces a relevant sociological term, namely recontextualisation, as the theoretical foundation in this present study. The internal relationship between different pedagogic discourses such as the Syllabus (EDB, 1999), the curriculum guideline (HKEAA, 2007), the examination paper (HKEAA, 2012) and the recommended mathematics textbooks (i.e.: Wong and Wong, 2009) are treated as part of the recontextualisation. A model of curriculum ecology is incorporated with the recontextualisation. Built on this model, the relationship between different pedagogic discourses emerging from one discipline and from an interdisciplinary approach can be modelled with recontextualisation as the driving force that mobilizes this model. Chapter 4 focuses on the research methodology chapter. It introduces the research paradigm, the research data, the justification for the research data, and the analytical models together with a sample of the research data. A blueprint for analysis is presented. Recontextualisation (Bernstein, 1990) as a sociological technical term is reconciled with reinstantiation (e.g. Martin, 2006; Hood, 2008). This reconciliation will provide recontextualisation with a linguistic model that can model the relationship between knowledge and representation from a linguistic standpoint. Chapter 5 details the analysis of the research data. Selected examples taken from the Syllabus (EDB, 1999), the curriculum guideline (HKEAA, 2007), the examination paper (HKEAA, 2012) and the textbook (Wong and Wong, 2009) are analysed with the help of reinstantiation. Chapter 6 discusses the findings of this study, concluding with Chapter 7 wherein conclusions are drawn, future research possibilities are postulated and possible limitations for generalisability foreshadowed.

Chapter Two
Literature Review of Mathematics: Its Knowledge Structure and Semiotic Resources

2.1 Introduction

The purpose of this chapter is to elaborate and compare the literature on the structure of mathematical knowledge and semiotic construction in mathematics. This chapter starts with a review of different perceptions of mathematical knowledge structure. In Section 2.2, the recommended organizational scheme of mathematical knowledge is provided, namely, the Conceptual Knowledge Structure by Hong Kong Examination Assessment Authority (HKEAA, 2007). In Section 2.3, I review the conceptual knowledge provided by Hebert and Lefebvre (1986). In Section 2.4, the sociological approach to knowledge construction by Bernstein (1999 and 2000) is outlined. Further to Bernstein's knowledge structure, how mathematical knowledge is conceived in the sociological tradition is provided in Sections 2.5 and 2.6 with the former based on Bernstein (1999 and 2000) and the latter based on Muller (2007). In Section 2.7, I summarize different approaches to knowledge construction and propose an approach that this study adopts. In the second part of the chapter, I review existing studies that underpin the correlation between mathematical knowledge and mathematical semiotic resources. Section 2.8 presents the procedural knowledge structure recommended by HKEAA (2007), Section 2.9 presents the procedural knowledge by Hebert & Lefevre (1986), Section 2.10 outlines Bernstein's way of describing language (2000) and its two descendant theories: namely, Grammaticality (Muller, 2007) in Section 2.11 and Semantic Density (Maton, 2009, 2011, 2011a & 2011b) in Section 2.12. Alongside the sociological approaches led by Basil Bernstein, research is also canvassed within the field of social semiotic theory. In Section 2.13, I introduce Halliday's (1978) view of mathematics. In Section 2.14, I discuss O'Halloran's (1996, 2000, 2005, 2007a, 2007b) contribution to language and mathematics. In Section 2.15, I summarize the existing research in this field and lead the review to an applicable analytical approach that is adopted in the present study to deal with the relationship between knowledge and representation. In the remainder of this chapter, I discuss the semiotic construction of mathematics. In Section 2.16, I review how mathematical language is recommended by HKEAA (2007) and its academic advisor (Mok, 2013) supplemented by a social semiotic recommendation informed by O'Halloran's work. In Section 2.17, I summarize this chapter.

2.2 Conceptual knowledge structure at HKDSE Level (HKEAA, 2007)

As suggested by the HKEAA, students in Hong Kong are equipped with the skills to "use appropriate mathematical techniques" (HKEAA, 2012, p. 2; 2013, p. 2) to answer questions and solve problems at the HKDSE level. A key component within these techniques is to interrelate "conceptual knowledge" (HKEAA, 2007, p. 104) amongst different and discrete mathematical concepts. Conceptual knowledge, as suggested by the curriculum guideline (HKEAA, 2007), portrays the relationship between mathematical concepts. The ideal capability of conceptual knowledge at HKDSE level is that the students demonstrate their competence in understanding how different mathematical concepts are connected with each other comprehensively and coherently. However, as reflected by the assessment report (HKEAA, 2012), students' competence of conceptual knowledge in mathematics needs improvement because students' overall performance is far from satisfactory. HKEAA suggests that in the examination, students only perform well when the mathematical concepts are "routinely learned" (HKEAA, 2007, p. 104) and procedurally organized. The notion of routine learning is expressed in terms of mathematical concepts to be examined being correlated to each other without much cross-referencing of concepts in different strands, or else they are organized in a stepwise manner. Conversely, once mathematical concepts are combined as different elements of knowledge, the overall performance of students tends to be unsatisfactory because they face difficulty in determining the type of mathematical knowledge that could be used for these questions (HKEAA, 2007, p. 104). This combination of different elements of knowledge is perceived as a complicated network of concepts within which the mathematical concepts examined, are implicitly correlated. The existence of a disjunction between concepts will result in the students' dissatisfaction, as their capability to uncover the connection between implicitly connected mathematical concepts is limited.

Therefore, as suggested by HKEAA (2007), the acquisition of *conceptual knowledge* will empower students with "a deeper understanding of mathematics" (p. 104) as the conceptual knowledge "helps them to make connections among different pieces of knowledge" (p. 104). What is more, as implied in suggestions by HKEAA (2007, p. 104), in general, the acquisition and application of the network of Mathematical knowledge among HKDSE candidates could be further developed and improved once the students consolidate and increase their competence in unravelling the complicated connection between different mathematical concepts.

2.3 Conceptual knowledge (Hiebert & Lefevre, 1986)

The conceptual knowledge conceived by HKEAA (2007) is adapted from Hiebert's (1986) edited work: *Conceptual and Procedural Knowledge: The case of Mathematics* that contains a series of articles dealing with knowledge construction within mathematics. According to Hiebert and Lefevre (1986, p. 5), "conceptual knowledge is characterized most clearly as knowledge that is rich in relationships. It can be thought of as a connected web of knowledge". This web of connection underlines the organizational relationship within mathematics where the conceptual knowledge is developed through the construction of "discrete pieces of information" (Hiebert & Lefevre, 1986, p. 5). The network of the linking processes could be further achieved through the connection "between two pieces of information that already have been stored in memory or between an existing piece of knowledge and one that is newly learned" (Hiebert & Lefevre, 1986, p. 5). These two processes of connection signify the prominence of a conceptual knowledge that highlights the status of the connection. Table 2.1 presents the components and functions within the conceptual knowledge of mathematics.

Table 2.1: The conceptual knowledge of mathematics (adapted from Hiebert & Lefevre, 1986, pp. 5–6)

Conceptual knowledge of mathematics		
Types of knowledge	**Components**	**Functions**
Conceptual knowledge	Discrete pieces of knowledge are connected in a network	Linking two already existing mathematical concepts Adding one newly learned piece of mathematical concept

Based on Table 2.1, in terms of the organizational structure, the connection between different mathematical concepts is utilized to examine the link between two existing concepts; this organizational structure could also be adopted to underpin how a newly learned concept is connected with an existing concept in the process of the acquisition of new concepts.

Initially, conceptual knowledge as depicted by Hiebert and Lefevre (1986) is designed to describe the underlying structure of the correlation between different mathematical concepts. This notion of organization echoes the intention as adopted in HKEAA's (2007) proposal within which the conceptual knowledge structure is available to unravel the organizational structure between different mathematical concepts. However, Hiebert and Lefevre's (1986) investigation penetrates deeper than the position held by HKEAA (2007). In Hiebert and Lefevre's (1986) understanding, conceptual knowledge incorporates both the correlation between discrete pieces of mathematical concepts and the representations of the mathematical knowledge. Concepts and representations have been described as a symbiotic unit that should be investigated synchronically. With regard to Hiebert and Lefevre (1986), in order to delineate the conceptual knowledge in detail, the relationship between two different mathematical concepts should be investigated with regard to:

1. How the concepts are portrayed (the relationship between mathematical concepts)
2. How the representations of the concepts are portrayed (the relationship between representations) and

3. How the representation visualizes the concept (the relationship between concept and representation).

Figure 2.1 visualises Hiebert and Lefevre's (1986) description of conceptual knowledge with reference to how the relationship between concept and representation is modelled with assistance from the "Primary level" and the "Reflective Level" (Hiebert and Lefevre, 1986, p. 4)

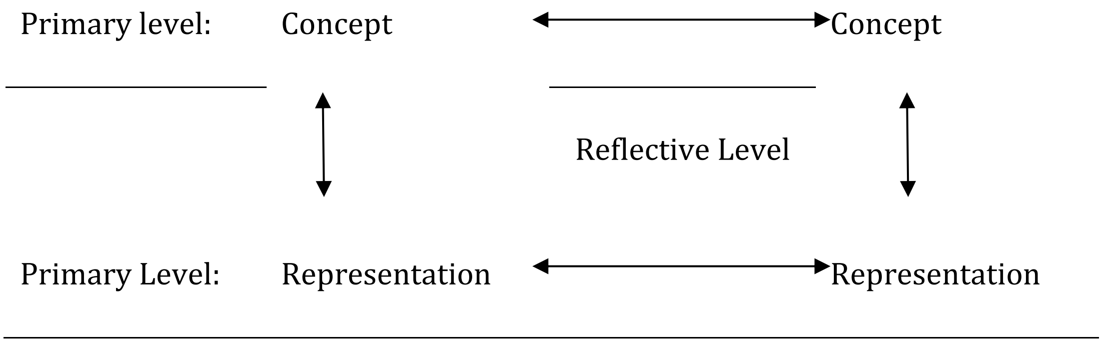


Figure 2.1: Visualized portrayal of conceptual knowledge with reference to Hiebert and Lefevre (1986)

In Figure 2.1, the primary level depicts the relationship between different mathematical concepts and the relationship between different representations. Comparable pairs on the same "primary level" (Hiebert & Lefevre, 1986, p. 4) are thought to have "the same level of abstractness" (1986, p. 5). For example, if we compare two mathematical concepts, they are thought to have the same level of abstraction. The same principle could be applied to compare two representations. The reflective level depicts the relationship between concept and representation, suggesting that representation exists in the material world.

Conceptual knowledge has evolved to include concept and representation. To decide the relationship between different types of mathematical knowledge, as Hiebert & Lefevre (1986) argued, is to determine the level of abstraction with respect to whether "a unit of knowledge (or a relationship) is tied to specific contexts" (Hiebert & Lefevre, 1986, p. 5). For instance, if we treat Pythagoras' Theorem as a mathematical concept, this mathematical concept could have different representations. Some representations are tied with specific contexts, such as the use of a bridge. For example, Figure 2.2 introduces a specific example of Pythagoras' Theorem. Some representations are not tied to specific contexts, such as using only a technical term. Although containing the same mathematical knowledge (Pythagoras' Theorem in this case), these two instances contain different conceptual knowledge with the one using a bridge being more context specific and the other not so.

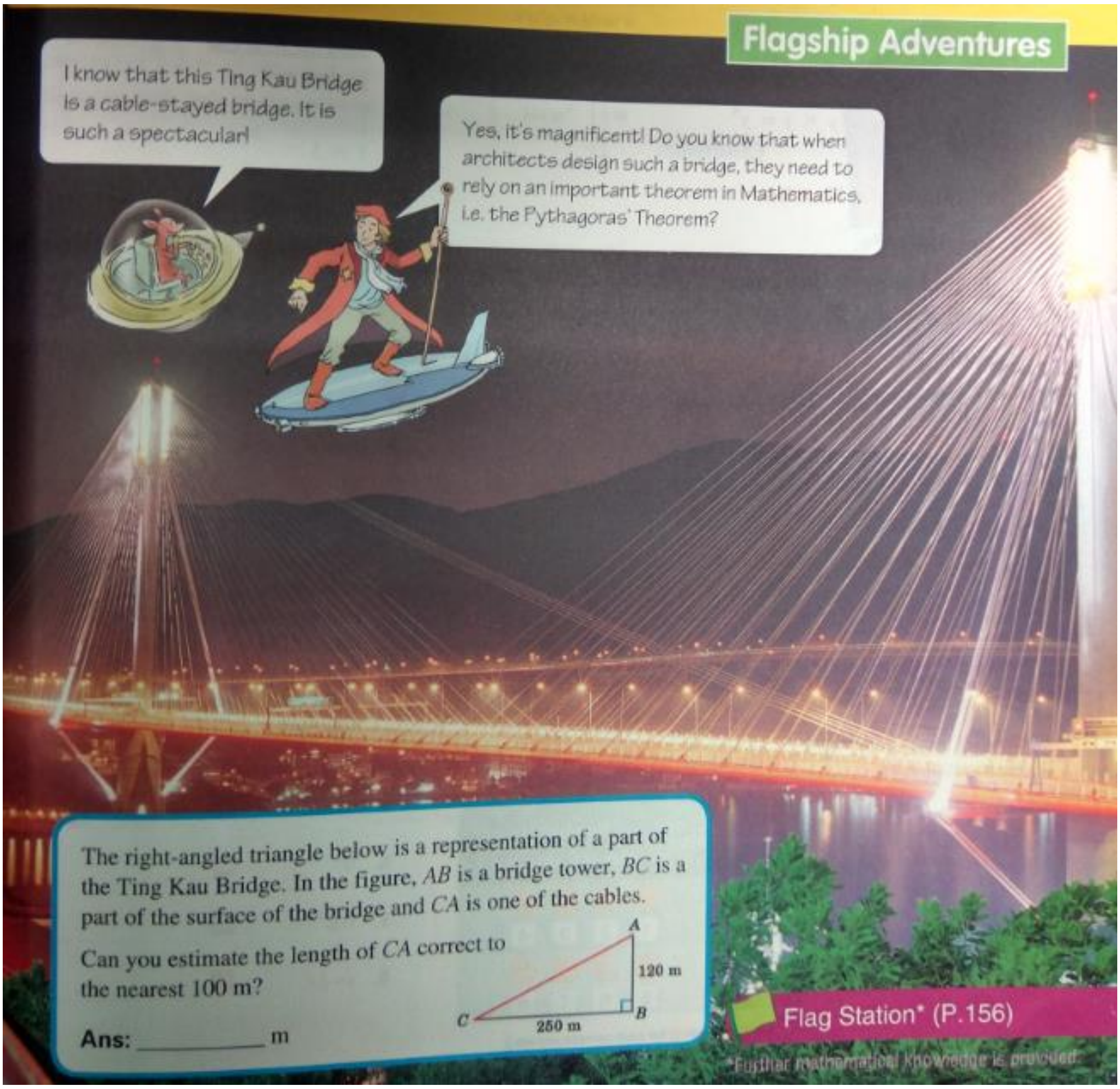


Figure 2.2: Example of Pythagoras' Theorem (Wong & Wong, 2009, p. 156)

In Figure 2.2, the introduction of Pythagoras' Theorem is projected in concert with the Ting Kau Bridge, named after an architect in Hong Kong. Different sides of the triangle labelled a surface of the bridge, a bridge tower and one of the cables. Drawing from Hiebert & Lefevre's (1986) level of abstraction, this example will become less abstract than when using technical data only, since the incorporation of a specific context related to a real bridge: the Ting Kau Bridge.

Although Hiebert & Lefevre's (1986) notion of conceptual knowledge is thought-provoking and inspiring, their treatment of mathematical knowledge as a blend of both concept and representation (for example, both the knowledge of Pythagoras' Theorem at conceptual world and the specific instances of Pythagoras' Theorem such as the Ting Kau Bridge are both mathematical knowledge at conceptual level requires more sophisticated work on the measurement of the level of abstraction for each piece of mathematical knowledge. Bearing in mind their contribution, treating mathematical concepts as a network, and ignoring their notion of the level of abstraction, this study suggests that from a purely conceptual perspective, mathematical concepts are organized as a network system. The relationship between different mathematical concepts could be interpreted based on Bernstein's (2000) notion of "horizontal knowledge structure" and "hierarchical knowledge structure". His work is able to incorporate and enhance the "conceptual knowledge" proposed by Hiebert and Lefevre (1986). Bernstein's perspective offers a refreshing focus on knowledge structure and could be utilized to discuss the inner structure of the "conceptual network of knowledge" (Hiebert & Lefevre, 1986, p. 4) with no need to consider the context and representation of mathematical concepts. Section 2.4 explores Bernstein's forms of knowledge and his description of the structure of mathematical knowledge.

2.4 Sociological approach to knowledge construction

In this section, Bernstein's classification and interpretation of the structure of knowledge is discussed. I begin with the forms of knowledge in the education field as outlined by Bernstein (1999 and 2000), before elaborating horizontal and vertical discourse. There are two distinctive, though correlated sub-categories within vertical discourse, namely the structure of horizontal knowledge and the structure of hierarchical knowledge, both reviewed with instances taken from the field of mathematics education.

2.4.1 Forms of knowledge in field of education

Bernstein (1999) proposed two opposed parameters in viewing the complicated phenomenon of knowledge construction in education. Horizontal discourse and vertical discourse are these two parameters, where "different forms of knowledge" (Bernstein, 1999, p. 158) are represented. Horizontal discourse is referred to as "everyday common-sense knowledge" (p. 158) while vertical discourse is referred to as "school(ed) knowledge" (p. 158). Both parameters describe the "invisible reality" (Bernstein, 2000, p. 16) of the structure of knowledge, specifying "the internal structure of specialised knowledges" (Bernstein, 1999, p. 157). However, both diverge from each other in their "sites of realisation" (Bernstein, 1999, p. 158).

2.4.2 Horizontal discourse: what takes place in daily life

Horizontal discourse is a form of knowledge that is "typified as everyday or 'commonsense' knowledge" (Bernstein, 1999, p. 159). The crucial feature of this type of knowledge is its segmental organisation (Bernstein, 1999, p. 159). In particular, horizontal discourse describes a phenomenon of isolation where "there is no necessary relation between what is learned in the different segments" (Bernstein, 2000, p. 159). That is to say, the relationship between different segments is arranged in a form lacking an internal relationship or in other words, isolated. Horizontal discourse therefore is concerned with the relationship within everyday knowledge where discrete segments of knowledge such as brushing teeth and riding bicycles are isolated. Both of these two segments according to Bernstein (1999, p. 162), are "locally" acquired in everyday life practices. These segments do not necessarily have equal status, and "clearly some will be more important than others" (Bernstein, 1999, p. 159).

2.4.3 Vertical discourse: what can be learned institutionally

In contrast with horizontal discourse, segments organized in the form of vertical discourse "are linked to others" (Bernstein, 2000, p. 160). The central feature of vertical discourse is that it is "official" (Bernstein, 1999, p. 162). By saying official, it means that vertical discourse is "supported institutionally" (Hasan and Butt, 2011, p. 106), as identified in the school curriculum.

Vertical discourse is concerned with how one particular curriculum builds up its own knowledge structure through interrelating different segments. The interrelation could be generalized in terms of three types: first, one segment integrates other segments; second, one segment parallels similar segments; and third, one segment contrasts with competing segments. The first internal connection is the characteristic of hierarchical knowledge structure and the other two internal connections are the characteristics of a horizontal knowledge structure. Examples from mathematics will be provided in Section 2.4.4 and Section 2.4.5.

2.4.4 The first dimension within vertical discourse: hierarchical knowledge structure

According to Bernstein (1999), hierarchical knowledge structure is one of the two organizational principles of knowledge construction involved in "official/institutional" sites. The organizational principle between different segments of knowledge within a hierarchical knowledge structure is that they are organized in a hierarchical and integrated manner. This manner is visualized in the shape of the layered triangle in Figure 2.3 inspired by the model provided by Bernstein (1999, p. 161).

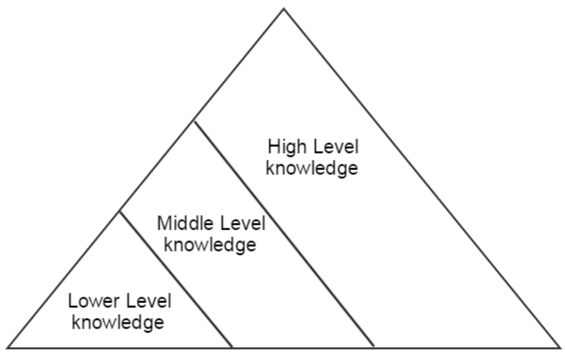


Figure 2.3: Hierarchical knowledge structure

The integration of knowledge represented in Figure 2.3, reflects a hierarchical knowledge structure that generates knowledge at a higher level of "abstraction, generality and integration" and could produce "progression in knowledge" (Moore, 2007, p. 50). Drawing from Bernstein's (2000) hierarchical knowledge structure, several studies have contributed to the discussion of how the knowledge structure within a certain subject is constructed. For example, disciplines within natural science (such as physics, chemistry and biology) are conceived as hierarchically structured. Adopting the genealogical organization of living things proposed by Haire et al. (2005, p. 202), Martin (2007) focuses on the knowledge structure in scientific taxonomies in biology. According to Martin, the biological taxonomies are organized hierarchically as they “classify and sub-classify at many levels of generality” (2007, p. 38). In Martin (2011), Bernstein’s (1999) hierarchical knowledge structure model is updated, suggesting the cumulative progression of a knowledge-building process. Two types of knowledge integration in Martin’s (2011) updates are illustrated in Figures 2.4 and 2.5, emphasizing the direction of knowledge progression and types of integration.

The first type shows that knowledge at a lower level is comprehensively integrated with the next highest level, and that accumulation is successively integrated with the highest level, as portrayed in Figure 2.4.

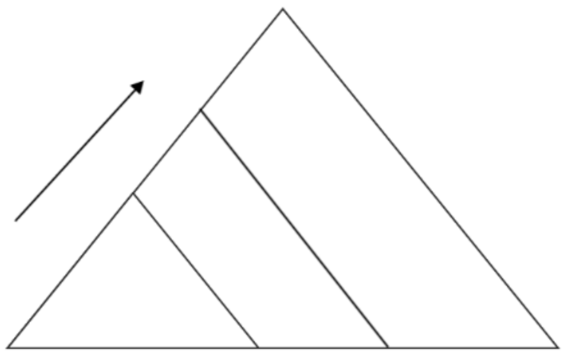


Figure 2.4: Hierarchical knowledge progression: comprehensive integration

Within this type of knowledge progression, the accumulation of knowledge at the upper level is achieved through the successive integration of knowledge from lower levels (Bernstein, 1999, p. 162). Essentially this visual portrayal resembles Bernstein’s (2000, p. 161) representation in Figure 2.3 with the addition of an arrow to signify the direction of knowledge progression from the least integrated level of knowledge to the most integrated level of knowledge. In short, the integration and progression is understood in terms of the relationship between prerequisite knowledge and required knowledge while the progression follows and is built upon the integration.

The other type of integration within hierarchical knowledge progression is partial absorption within which part of the knowledge at the lower level was integrated in the upper knowledge levels. Figure 2.5 visually portrays this type of knowledge integration.

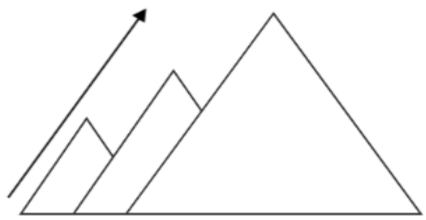


Figure 2.5: Hierarchical knowledge progression: partial integration

This partial integration in the progression of hierarchical knowledge was introduced by Martin (2011, 2014) and thought to be the most common form of knowledge progression in education contexts.

Lindstrøm (2011) provides a model capturing both comprehensive knowledge progression and partial knowledge progression specific to physics, as portrayed in Figure 2.6.

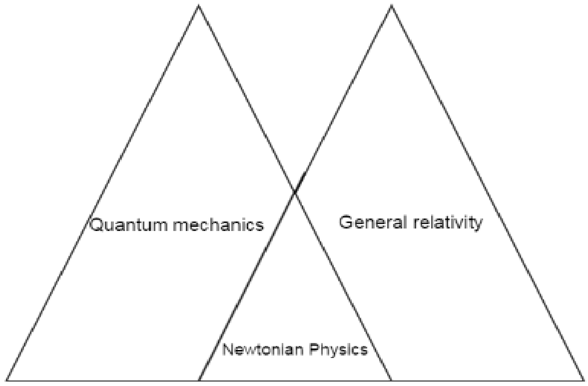


Figure 2.6: Hierarchical knowledge structures in physics: Relationship between Newtonian physics and modern fields of quantum mechanics and general relativity (adapted from Lindstrøm, 2011)

Figure 2.6 suggests the relationship between three physical concepts: Quantum mechanics, Newtonian Physics and General relativity. Inspired by Figures 2.3 to 2.5, where the models of progressive hierarchical knowledge structures have been sequentially presented, this Figure 2.6 also suggests the hierarchical knowledge construction between three physical concepts. In other words, Quantum mechanics comprehensively integrates and subsumes Newtonian Physics as part of its own knowledge structure. A comprehensive integration is similarly identified between General relativity and Newtonian physics with the latter as a part of the former. The relationship between Quantum Mechanics and General relativity is also depicted as a form of hierarchical organization. Drawing from Martin's (2011) model of partial integration between different areas of scientific knowledge in Figure 2.5, Quantum Mechanics is partially integrated with General relativity with their area of commonality being Newtonian physics.

It is interesting to mention here, that although Newtonian physics is the stem of modern physics, it also integrates ancient Greek science such as Euclid's Elements. Therefore, a famous proverb from Newton, "If I have seen further, it is by standing upon the shoulders of giants" (cited by the British Broadcast Corporation, 2014) depicts the basic steps in establishing the new theories in natural science whose major achievements are established through the integration of previous work. This integration is classified as a hierarchical knowledge structure, in Bernstein's (1999) term.

2.4.5 Second dimension within vertical discourse: horizontal knowledge structure

In opposition to the hierarchical knowledge structure, the other type of knowledge structure within vertical discourse is termed “horizontal knowledge structure” consisting of “a series of specialised languages with specialised modes of integration and criteria for the construction and circulation of texts” (Bernstein, 1999, p. 162). Within a horizontal knowledge structure, different segments of knowledge embody “their distinctive and incommensurable sets of experience” (Moore, 2007, p. 50). These sets of experience are “not translatable” because each set has its own “criteria for legitimate text” (Bernstein, 1999, p. 162). The development of knowledge within a horizontal knowledge structure is through “the introduction of a new language” (Bernstein, 1999, p. 163) which is internally associated with the existing language as both of the languages could be derivatives from a higher category but independent of each other. Language here is understood as the segment of knowledge that has the potential to evolve and to be in connection with other languages (other segments of knowledge). This connection which is not like integration in hierarchical knowledge structure, introduces a new segment of knowledge which could be interpreted as either opposite to each other and resembles the relationship between the two sides of one coin, or correlated but incommensurable as each segment realizes “a fresh perspective” (Bernstein, 2000, p. 162). There is an independency between different segments in horizontal knowledge structure.

This knowledge construction structure suggests that the interrelation between one type of knowledge and the other is “bound up” with a distinctive focus and does not have any form of integration (Bernstein, 2000, p. 162). The internal relation between the knowledge that is correlated horizontally is that they are on the same level of abstraction and do not overlap with each other. In Bernstein’s visualized portrayal (1999, p. 162), the horizontal structure of knowledge is as illustrated in Figure 2.7:

$L^1L^2L^3L^4L^5L^6L^7\ldots L^n$

Figure 2.7: Horizontal knowledge structure (Bernstein, 1999, p. 161)

As shown in Figure 2.7, within the horizontal knowledge structure, different segments of knowledge are correlated but inconsonant with each other. They are correlated in the sense that unlike segments organized in horizontal discourse, different pieces of knowledge within a horizontal knowledge structure are grouped together under a higher category but inconsonant to each other. Metaphorically speaking, different segments of knowledge within a horizontal knowledge structure are like the siblings within a family. They share the similar DNA as determined by their family genetic structure, but develop as inconsonant individuals with their own characteristics and personality.

Figure 2.7 highlights one of the typical features of a horizontal knowledge structure that is the generation of new segments of knowledge by adding new and fresh perspectives to that loop. The other typical feature mentioned before is that segments within a horizontal knowledge structure resemble the relationship between the two sides of one coin. Martin (2011) suggests that this type of organization could be visualized as a classic Chinese "yin and yang" structure with each side opposing the other. Figure 2.8 is the other type of horizontal knowledge structure drawn by Martin (2011).

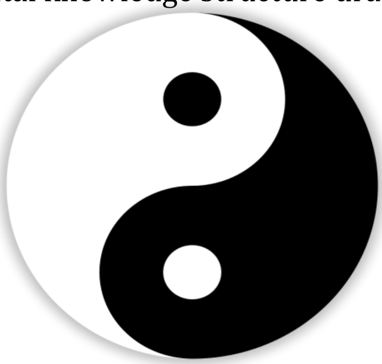


Figure 2.8: Horizontal knowledge structure in "yin and yang"

Figure 2.8 shows that different segments of knowledge are in opposition to each other and could be interpreted as antinomic from a linguistic perspective.

Within the education field, as argued by Bernstein (2000), "horizontal knowledge structures" are labelled as the knowledge structure for "English Literature, Philosophy and Sociology" (p. 161), "Linguistics and Economics" (p. 163) and "Mathematics" (p. 163). To exemplify his notion of "horizontal knowledge structure" into concrete instances, several ideological theories in Sociology, namely "functionalism, post-structuralism, post-modernism, Marxism, etc." were elaborated to support this argument of the intrinsic feature of the horizontal knowledge structure. Within this structure, different sets are "not translatable" with their own "particular favoured or originating speakers" (Bernstein, 2000, p. 162). Based on Bernstein's (2000) classification, the subject of history which is elaborated by Martin (2007), is intended to "be instantial" (p. 42) with new entities "arising in the course of the development of a particular discussion" (p. 42) through activity and chronological sequence. These new entities that have been "technicalized" as the participants in one historical event, through the process of "Thingification" (Martin, 2007, p. 45), are paralleled with each other without integration. For instance, the four waves of Indochinese migrations into Australia recorded by Brennan (2003, p. 29 & p. 31) are categorized as four independent but chronologically related events by Martin (2007, p. 44). These four waves are organized within a horizontal knowledge structure. As concerned by Bernstein (2000) and reinforced by other scholars, history displays the fundamental and typical characteristics of a horizontally organised curriculum.

The review suggests that Bernstein's (1999, 2000) conceptual frameworks are now undertaken by scholars in understanding knowledge construction in the education field (for example, see Muller et al., 2004; Christie and Martin, 2007; Ivinson et al., 2011). Corresponding with the emerging research following the work by Bernstein, Christie (2007) believes that Bernstein's description of the construction of knowledge is "a theory of knowledge structure" (p. 7) and this theory is valuable and able to be adopted in the education field.

The next section will narrow down the review from the theoretical consideration offered by Bernstein and others to the specific field of mathematics. Drawing from the horizontal knowledge structure and the hierarchical knowledge structure, the knowledge structure of mathematics will be considered.

2.5 Bernstein's understanding of mathematical knowledge structure

The above detailed description of the horizontal knowledge structure and the hierarchical knowledge structure, consolidates the theoretical foundation through which the knowledge structure in natural science subjects and humanity subjects could be modelled, as drawn from Bernstein's (1999, 2000) understanding of the knowledge structure.

It is my intention to suggest that Bernstein's classification of the knowledge structure proves influential and informative. For certain subjects, Bernstein (2000) himself has already explicated their knowledge structures, incorporating his insights into the field of education. For example, he portrays natural science subjects (e.g. physics, chemistry and biology) as hierarchical knowledge structures, while linguistics, literature and history are portrayed as horizontal knowledge structures.

His contribution is meaningful as it unravels the construction of knowledge and considers knowledge itself.

Because the major focus of the present study is to investigate the disciplinary knowledge of mathematics, it is worth mentioning here how Bernstein understood the knowledge structure of mathematics beforehand. Bernstein (2000) highlighted that the mathematical knowledge structure is constructed in the form of a horizontal knowledge structure, because mathematics "consists of a set of discrete languages for particular problems" (p. 163). Bernstein (2000) believes that the organizational system for mathematics is constructed horizontally; for example, discrete segments such as, algebra, statistics, geometry and other specialized fields in mathematics are inconsonant with each other. This inconsonance signifies that the knowledge structure of mathematics is a horizontal knowledge structure.

2.6 Extending Bernstein's approach: A Bernsteinian scholar's understanding of mathematical knowledge structure

Being a Bernsteinian scholar, Muller utilizes Bernstein's vertical discourse to explore mathematics. Muller's (2007) description of the mathematical knowledge structure differs from Bernstein's descriptions. In Muller's (2007) opinion, he confirms that Bernstein's notion of the knowledge structure in the field of education is applicable through working on the curriculum structure of mathematics, linguistics and sociology. However, Bernstein's (2000) description of mathematical knowledge is reinterpreted by Muller (2007). He argues that "verticality of a kind approaching the triangular form obtained in hierarchical knowledge structures" (p. 70), is a combination of both horizontal knowledge structure and a hierarchical knowledge structure. Muller's (2007) work implies that Bernstein's (2000) description on the knowledge structure of mathematics could be further developed when adapted to mathematical knowledge construction. Horizontal knowledge structure where "a set of discrete languages" (Bernstein, 2000, p. 161) is instantiated, is proposed as the relationship between independent fields of education study such as the relationship between algebra, geometry and statistics. In contrast, mathematics also takes the form of hierarchical knowledge structures in constructing the knowledge where new knowledge "integrates knowledge at lower levels" (Bernstein, 2000, p. 161). For example, trigonometry is a part of geometry. Drawing from the work by Muller (2007), Mathematical knowledge structure is constructed as a hybrid of both hierarchical and horizontal knowledge structures.

2.7 Summary of mathematical knowledge structure: a comparative perspective and recommendation

In mathematics, knowledge as conceived in the present study should be regarded as a concept rather than the constellation of both concepts and instantiation, which is the position held by Hiebert and Lefevre (1986). The reason to view knowledge as concept is threefold. First, it offers space for flexibility as mathematical knowledge is treated as a mathematical concept regardless of its physical representations. Second, it enables a correlation among different mathematical concepts to be portrayed at the same level of abstraction, highlighting the network structure to be imposed onto different mathematical concepts. Third, for analytical purposes, the differentiation of a mathematical concept and a mathematical representation, enables them to be examined differently in later work.

In terms of the mathematical knowledge structure inherited from Bernstein's (1999 and 2000) works on the knowledge structure, this present study favours Muller's (2007) description of mathematical knowledge structure. Mathematical knowledge structure is viewed as both hierarchically organised and horizontally organized. The motivation behind this clarification is that once the mathematical knowledge structure is portrayed in two dimensions, a correlated network could be displayed with an illustration of the connection between different mathematical concepts. With the help of this network, the progression of knowledge could be visualized either in the form of interpretation as in a hierarchical knowledge structure, or in the form of paralleling and opposition as in the horizontal knowledge structure. In my opinion, the mathematical knowledge structure is a conceptual network with which different mathematical concepts are correlated in the form of paralleling, opposition and interpretation.

To synthesize the above discussion, what has been addressed is the organisation of mathematical concepts and the relationship between mathematical concepts that could be mapped onto a network whose organizational principles are both a horizontal knowledge structure and a hierarchical knowledge structure. The above discussion is primarily concerned with the conceptual factors, leaving the discussion regarding the relationship between concept and its representations, to the remainder of the chapter.

2.8 Representation of knowledge considered by Hong Kong Examination and Assessment Authority (2007)

As has been reviewed in Section 2.2, the conceptual knowledge at HKDSE level is a network in which different mathematical concepts are connected. The network of mathematical concepts is invisible (Devlin, 1998). This network needs translation into mathematical language that is visible in reality. The appropriate way of structuring mathematical language is "to communicate ideas and to present arguments mathematically", organising its component "numbers, symbols and other mathematical objects", clearly and logically (HKEAA, 2007, p. 2).

2.9 Representation of knowledge considered by Hiebert and Lefevre (1986)

The representation of mathematical knowledge in visible reality includes "the formal language, or symbol representation system, of mathematics" (Hiebert and Lefevre, 1986, p. 6). Formal language is interpreted as verbal written texts that are supposed to be grammatically correct, transmitting meaning that will not be misunderstood. The symbol representation system is a system of mathematical symbols that function as substitutes for verbal language, correlating with verbal language to create meaning and express information. In the interpretations by Hiebert and Lefevre (1986), verbal language and mathematical symbols are the basic components of mathematical language. Mathematical meaning is expressed with the help of verbal language and mathematical symbols. The combinations of verbal language and mathematical symbols could result in the introduction of a mathematical term, the elaboration of a mathematical issue and the answering of a mathematical question, and the like. These different combinations may be interpreted as the semiotic examples of mathematical language.

In terms of the organizational principles for these semiotic combinations of mathematical language, "algorithms or rules" (Hiebert and Lefevre, 1986, p. 6) are proposed as the operational regulations that enable the progression of meaning in mathematics. Algorithms or rules complete mathematical tasks in "a predetermined linear sequence" (Hiebert & Lefevre, 1986, p. 6). Their function is to express the sequence of mathematical meaning linearly. This is predetermined by Hiebert and Lefevre (1986), that the solution of the mathematical tasks is presented and depicted based on "step-by-step instructions" which "prescribe how to complete tasks" (p. 6). Hence, the linear order within mathematics is described as the procedure assigned to the problem solving and concept explanation. This procedure is orderly, routine and often predesigned.

2.10 Bernstein's internal and external language of description

Bernstein (2000) proposed horizontal discourse and vertical discourse to indicate the disciplinary knowledge construction. Within vertical discourse, the differentiation of horizontal knowledge structure and hierarchical knowledge suggests knowledge construction within a certain subject. In order to discuss how his notion of invisible knowledge structures works, Bernstein (2000) proposed the notion of “languages of description” (p. 132). The nature of invisible knowledge structure and the representation of knowledge in the real world are described as different “languages of description” (Bernstein, 2000, p. 132). Two distinctive subsets: “internal language of description” (labelled, L^1 by Bernstein) (p. 133) and “external language of description” (labelled, L^2 by Bernstein) (Bernstein, 2000, p. 133), are rendered to describe knowledge and its representation with different functionalities. L^1 is concerned with the internal knowledge structure that is invisible, while L^2 is concerned with translating the invisible L^1 into visible realities. Therefore, as Bernstein (2000) suggests, “internal languages are the condition for constructing invisibles, external languages are the means of making those invisibles visible, in a non-circular way” (p. 133). He also underlines the principles in depicting these two languages of description, as “the external language of description (L^2) is the means by which the internal language (L^1) is activated” (Bernstein, 2000, p. 133).

2.11 Muller’s grammaticality

Following Bernstein’s (2000) two languages of description, Muller (2007) emphasizes “the external sense of grammaticality” by suggesting that “grammaticality (in the external sense) has to do with how theory deals with the world, or how theoretical statements deal with their empirical predicates” (Muller, 2007, p. 71). Grammaticality in Muller’s work corresponds with Bernstein’s external language of description, dealing with “a specific text” (Bernstein, 2000, p. 133). Knowledge is reflected as “empirical referents” (Bernstein, 2000, p. 133) in the external sense. These empirical referents relate with each other and produce a specific text. With respect to the organising principle, “the stronger the (external) grammaticality of a language, the more stably it is able to generate empirical correlates and the more unambiguous because more restricted the field of referents” (Muller, 2007, p. 71).

2.12 Maton's semantic density

Muller's efforts in determining the ambiguity level of "empirical referents" for invisible knowledge inspired Maton's Semantic Density (2011). The notion of Semantic Density was proposed by Maton to refer to the degree of meaning condensation within practices (Maton, 2011 & 2014). For instance, the strengthening of Semantic Density is the process of condensing a lengthy description into a technical term, and the weakening of Semantic Density is the process of unpacking an abstract idea back into a lengthy description or specific examples (Maton, 2011). Semantic Density is proposed to measure the condensation level of meaning (e.g. Maton, 2011, 2013, 2014). The notion of Semantic Density is an upgraded format of Grammaticality. First, Semantic Density visualizes the condensation level of meaning in a coordinate. The greater the meaning is condensed the higher the Semantic Density it bears. Second, Semantic Density always works together with Semantic Gravity, which is used to measure the degree to which meaning is related to its context (Maton, 2011). These two newly proposed parameters collaborate to formulate the notion of Semantic Waves and Semantic Scale that are used to describe how meaning is unpacked and repacked by teachers in classrooms (Maton, 2011). The model of unpacking and repacking of knowledge is provided in Figure 2.9.

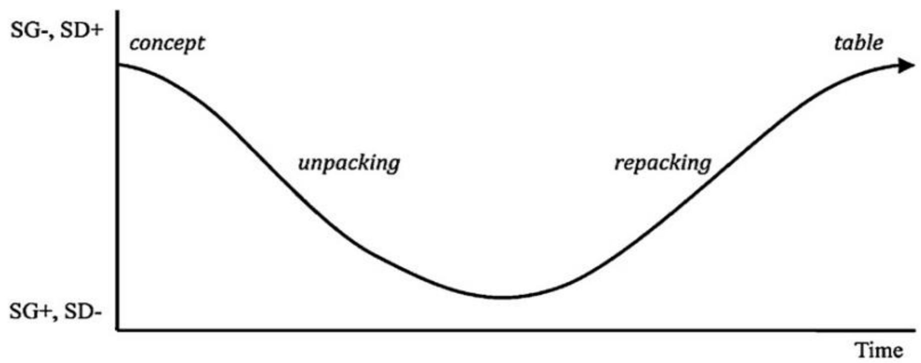


Figure 2.9: Unpacking and repacking of knowledge in teacher talk (Maton, 2011)

This model suggests the annotation of how one specialised terminology is taught in classroom. The teacher starts with a technical term in the beginning, and then explains the term with lengthy description. At the end of teaching, the teacher restates the technical term. Drawing from Semantic Density, technical term is more semantically denser than a lengthy description since the same amount of meaning is encapsulated into a relatively smaller unit. That one technical term is relatively smaller than a lengthy description, is because of the use of less wording in the technical term. Drawing from Semantic Gravity, a technical term is less contextually dependent than a lengthy description, since the technical term is more flexibly reproduced and relocated into other contexts. Equally, for lengthy description, its contextual dependence level is higher than that of a technical term because it cannot be relocated into other contexts as easily as a technical term.

Maton (2011) termed this model a Semantic Profile, the transition of meaning from abstract to the concrete and ending at the abstract level.

Maton's work on the external language of description is largely argued from a notational perspective. It has not been thoroughly worked out to offer a numeric formulate that could precisely calculate the Semantic Density and Semantic Gravity. However, studies that offer linguistic explanations for Maton's sociological interpretation of the relationship between knowledge and representation are now emerging (e.g. Martin, 2015). When consolidated with linguistic explanations, Maton's work may be interpreted more effectively.

2.13 Social semiotic construction of knowledge: knowledge and the representations of knowledge (Halliday, 1978; Halliday & Matthiessen, 1999)

Inherited from Saussure's (1959) work on language and meaning, semiotic resources are resources that are capable of making meaning and carrying meaning (Halliday, 1978; Matthiessen, forthcoming). A social semiotic approach in viewing knowledge and representation is to separate these two entities from each other in the first place, and then concentrate on how representations inform, and are informed by, knowledge in socially constructed contexts. An artificial, though clear-cut, division separates knowledge and representation allowing their symbiotic relationship to be investigated chiefly from the representation side. On the one hand, the physical quality of knowledge is invisible in essence, and this invisibility of knowledge needs to be "expressed in one way or another through the medium of words and structure of a language" (Halliday, 1978, p. 197). On the other hand, through language, we could deal with the knowledge. Corresponding to Halliday and Matthiessen's (1999) argument, language serves as the visible representation of invisible knowledge because "all knowledge is constituted in semiotic system" (p. 3).

This social semiotic approach (Halliday, 1978) resembles the sociological perception (Bernstein, 2000; Muller, 2007) which labels knowledge as "internal" while representation as "external" (Bernstein, 2000, p. 123) forms a dichotomy between knowledge and representation. Insights taken from the semiotic approach in viewing the relationship between knowledge and language enriched the sociological perception with the provision of a linguistic analysis in viewing how knowledge has been represented in the real world. The dialogue between these two scholarly schools includes the work undertaken collaboratively by Bernstein, Hasan and Halliday in the late 70s (e.g. Hasan 1973), edited work by Martin and Christie (2007), Christie, Martin and Maton (2011) and a special issue in Linguistics and Education (2011, vol 1). A comprehensive review of how these two streams have co-developed in the past four decades is documented in Martin (2014).

A social semiotic perspective displays the explanatory power of linguistic phenomena in socially constructed contexts, and how they could be theorized. This explanatory power considers language as the primary semiotic resources in which knowledge could be represented in the real word.

Language as the primary meaning-making resource incorporates Saussure's (1959) notion of "Semiosis", leading to Halliday and Matthiessen's (1999) proposal that knowledge is "modelled on natural language in the first place" (p. 25). Language as considered by Saussure (1959), Halliday (1978), Halliday and Matthiessen (1999) departs from a traditional linguistic perceptive which departs from verbiage such as words, clauses and clause complexes for analysis. With the development of multi-semiotic studies, language has incorporated semiotic resources more than verbiage alone. Informed by Halliday's social semiotic theory in particular, the development of multi-semiotic studies has theorized how various semiotic resources make meaning and carry meaning, including picture, colour, music, display art, film and so on.

Within the realm of mathematics, the representation of mathematical knowledge is achieved through mathematical discourse. Mathematical discourse is in turn organized by mathematical semiotic resources such as verbal language, mathematical symbolism and visual images. Therefore, following the social semiotic perspective of knowledge building, the representation of the disciplinary knowledge of mathematics is represented into mathematical semiotic resources.

2.14 O'Halloran's work on mathematical discourse and multimodal grammaticality

Developing from Halliday's Lexicogrammatical Systems (Halliday and Hasan, 1976; Halliday, 1994; Halliday and Matthiessen, 2004) and Martin's Discourse Semantic Systems (Martin, 1992; Martin and Rose, 2003), O'Halloran who is "the main specialist in SFL and the teaching of Mathematics" (ISFLA Website Comments, 2004) has worked on the interpretation of the meaning-making process in mathematics from a social semiotic perspective.

O'Halloran's work on mathematical discourse is built upon Martin's (1992; Martin and Rose, 2003) discourse semantic system. Martin's work extends Halliday's concerns regarding lexicogrammatical features and their function at the rank of word and clause and treats text as the unit of analysis. According to O'Halloran (2005), Martin's (1992) discourse semantic system "leads to a language plane with two strata, discourse semantics and lexicogrammar" (p. 63) and "is useful for the analysis of stretches of text which involve language, visual image and mathematical symbolism" (p. 65). O'Halloran (2005) reworked Martin's frameworks to enable a discourse semantic system to account for the multi-semiotic phenomena that prevails in mathematical discourse. Her reworked framework theorizes how meaning is expressed "within and across different semiotic resources" (O'Halloran, 2005, p. 65) by conceiving the meaning-making process in terms of two parameters: "intrasemiosis in linguistic language, mathematical symbolism and visual images" as the first parameter, and "intersemiosis across the three semiotic resources" as the second parameter (O'Halloran, 2005, p. 65). These two parameters have been elaborated in detail in O'Halloran (2005) from a meta-functional perspective to see how mathematical semiotic resources are coordinated experientially, logically, interpersonally and textually. This is considered not only at the lexicogrammatical level (Halliday, 1994) but also at the discourse level (Martin, 1992; Martin and Rose, 2003), when the mathematical discourse involves the unfolding of stretches of text. O'Halloran's (2005) mathematical discourse models the orchestration of mathematical semiotic resources at the discourse strata and gives rise to a "result in the 'texture' of a text" (O'Halloran, 2005, p. 66) in multi-semiotic environments.

With reference to how mathematical knowledge is constructed in mathematical discourse, and building upon Muller's grammaticality in an external sense (2007), O'Halloran (2007a) proposed the multimodal grammaticality to include multi-semiotic phenomena of mathematical knowledge construction, for consideration.

Muller's work has been positively evaluated by O'Halloran (2007a) as she believes that Muller's grammaticality is "a significant step" in helping us understand the knowledge-building process as it provides "an increasing explanatory power" (2007a, p. 212) to the external world. Relying on Halliday's (1978) social semiotic theory, O'Halloran (2007a, 2011) has upgraded Muller's grammaticality. She creates the notion of "Multimodal Grammaticality" (O'Halloran, 2007a, p. 215) to investigate the accumulation and the construction of mathematical knowledge when different semiotic resources are involved. As has been reviewed in the above section, semiotic resources in mathematics include verbal language, mathematical symbolism and visual imagery (2005, 2007a, 2011).

Compared with her work on mathematical discourse (O'Halloran, 2005), the central focus of multimodal grammaticality has shifted from describing the multi-semiotic phenomena to capturing how mathematical knowledge has been made visible (O'Halloran, 2007a, p. 215). The concept of the "visibility" of knowledge

(O'Halloran, 2007a, p. 215) is proposed. Visibility is gradable and could be measured on a scale that is transparent and explicit at one end, and vague and implicit at the other. The levels of visibility possessed by different combinations of mathematical semiotic resources are concerned with different processes of visualization of mathematical knowledge. These processes of visualization are the empirical referents of mathematical knowledge as in sociological terms. Multimodal grammaticality deals with "the individual and integrated functionality" (O'Halloran, 2007a, p. 215) within and across different empirical referents, such as the instances of knowledge representation.

Each empirical referent as one instance of knowledge representation is considered from a multi-semiotic viewpoint. This viewpoint considers how mathematical semiotic resources such as "verbal language, mathematical symbolism and visual imagery" (O'Halloran, 2007a, p. 214) function independently and interactively in the construction of mathematical knowledge. In this present study, the following steps are proposed to work on the visibility for each instance of knowledge representation:

- How is mathematical knowledge represented in the real world?
- What semiotic resources are utilized to construct these real world representations?
- What will be the meaning potential of these semiotic resources?

The foregoing steps are ways of considering mathematical knowledge construction from a social-semiotic perspective.

2.15 Mathematical language and mathematical discourse: What does the Hong Kong Education Bureau believe?

According to the Hong Kong Curriculum Council, mathematical language includes "graphs, figures and symbols" (p. 39). With reference to EDB (1999, p. 4) and HKEAA (2007, p. 30), mathematical language is composed of "numbers, symbols and other mathematical objects" (EDB, 1999, p. 4; HKEAA, 2007, p. 30). As suggested by the academic advisor of the Professional Development Programme of the Hong Kong Special Administration Region, Dr Ida Mok (2013), mathematical language comprises “vocabulary, symbols, representations, text” (2013, p. 2).

The function of mathematical language is to “analyse and present possible solutions to a problem and discuss with others” (Hong Kong Curriculum Council, 2000, p. 39) and to “communicate with others and express their views clearly and logically” (HKEAA, 2007, p. 125).

Within the realm of mathematical education, mathematical language is used to present arguments mathematically, using symbols such as “$2×3” and graphs “like bar graphs” (Curriculum Council, 2000, p. 38). With reference to other subjects, mathematical language will also be utilized to represent “laws and formulae” (Curriculum, Council, 2000, p. 18; HKEAA, 2007, p. 36). For example, the classic Newton’s Law of Motion, F=ma, is represented in mathematical symbolism as “=”, and so on.

According to the EDB, the distinctive feature that differentiates mathematical language from verbal language is the use of graphs, figures, symbols, numbers and other mathematical objects. The method the EDB adopts is enumeration. However, the list is not exhaustive so EDB provides a term “other mathematical objects” (EDB, 1999, p. 4; HKEAA, 2007, p. 30) to suggest the remaining instances in mathematical language.

According to Mok (2013), mathematical discourse comprises oral and written text organized in mathematical language such as “vocabulary, symbols, representations, text” (p. 2).

A diagrammatic representation differentiating between mathematical discourse and mathematical language and its components, as perceived by EDB, is portrayed in Figure 2.10.

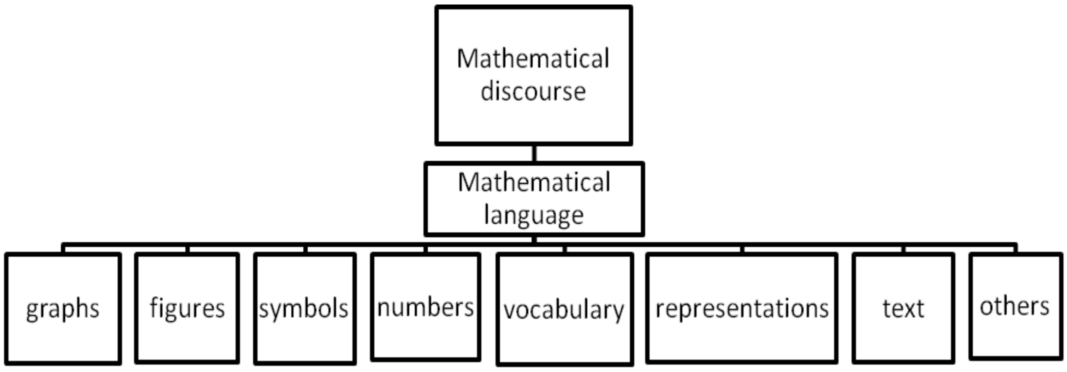


Figure 2.10: Relationship between mathematical discourse, mathematical language and the components of mathematical language

The relationship as presented in Figure 2.10 suggests that Mathematical discourse is composed of mathematical language such as graphs, figures, symbols, numbers, vocabulary, representations, text and other mathematical objects as explicated by EDB (1999), the Hong Kong Curriculum Development Council (HKCDC) (2000), HKEAA (2007) and Mok (2013).

Mathematical semiotic resources considered by EDB (1999), CDC (2000), HKEAA (2007) and Mok (2013), are all composed from an enumerative perspective. Mathematical semiotic resources are enumerated as separate items. A theorized theoretical contribution will be provided in 2.16 where mathematical semiotic resources are categorized according to their specific natures.

2.16 Mathematical language and mathematical discourse: a social semiotic perspective

Drawing on Halliday's systemic functional model of language (1978), O'Halloran (1996, 2000, 2005, 2007a, 2010 & 2014) discussed the mathematical language from a social-semiotic perspective where "mathematics is not construed solely through linguistic means" (O'Halloran, 2000, p. 360). Mathematical language, as considered by O'Halloran, is construed through the semiotic resources of language, mathematical symbolism and visual images.

2.16.1 Verbal Language

Verbal language is one of the semiotic resources in mathematical discourse. Halliday, (1978) suggested that mathematical meaning is expressed through "the mathematical use of natural language" (p. 195). The linguistic features of mathematical language have been investigated in terms of "the technical vocabulary" (MacGregor, 2002), "dense noun phrases in clauses and sentences" (Sfard & Lavie, 2005), and "frequent use of conjunctions" (Schlepegrell, 2007). These linguistic aspects of mathematics echo with Halliday's (1978) notion of a "mathematics register" where mathematical language should be a language that is used for "mathematical purposes" (p. 195). Overall, the mathematical purposes (Halliday, 1978, p. 195), could be achieved through the "language", because within the realm of mathematics, "language is used to reason about the mathematical results in a discourse of argumentation in which mathematical processes are related to each other and interpreted" (O'Halloran, 2014, p. 9). For example, technical taxonomy and technical terminologies (i.e. Pythagoras' Theorem) are created by "using language" (O'Halloran, 2014, p. 10).

Various studies (e.g. MacGregor, 2002; Sfard & Lavie, 2005) on the role of language performed in mathematical language have been synthesized by Schlepegrell (2007) who proposed the linguistic challenges following a review. She suggested that "the multi-semiotic formations of mathematics" (p. 139) should be highlighted to account for the reality that the "mathematics register draws on a range of modalities, constructing meaning by deploying multi-semiotic resources" (Shlepegrell, 2007, pp. 146–147). As argued by O'Halloran (2010), "Mathematics is not construed solely through linguistic means. Rather, mathematics is construed through the use of the semiotic resources of mathematical symbolism, visual display in the form of graphs and diagrams, and language" (p. 360).

2.16.2 Mathematical symbolism

Mathematical symbolism or "mathematical symbolic notation" (O'Halloran, 2014, p. 4), is the phenomenon where "key quantities and processes were symbolized in the linguistic text to the contemporary symbolic form" (O'Halloran, 2014, p. 4).

The "mathematical participants" and the "mathematical process" (O'Halloran, 2010, p. 221) will be simplified so that "the symbolism becomes a specialised tool for logical reasoning" (O'Halloran, 2010, p. 221). For example, Figure 2.11 is an explanation of "Pythagoras' Theorem" given by Wong and Wong (2009, p. 103).

In a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of two adjacent sides.

i.e. In ΔABC, if $\angle C=90°$,

then $a^2+b^2=c^2$.

(Abbreviation: Pyth. theorem)

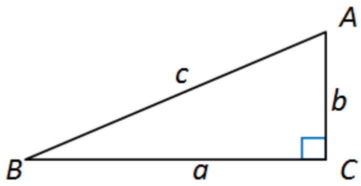


Figure 2.11: Multi-semiotic explanation of Pythagoras' Theorem (Wong & Wong, 2009, p. 103)

The relationship between the linguistic explanation is that "in a right-angled triangle, the sum of the squares of the two legs is equal to the square of the hypotenuse" and the symbolic explanation: "In ΔABC, if ∠C=90°, then $a^2+b^2=c^2$" is that the complete relationship between hypotenuse and two legs in the right-angled triangle is encoded. "Right-angled triangle" has been encoded as "In ΔABC, if ∠C=90°", "the sum of the squares of the two legs is equal to the square of the hypotenuse" has been encoded as "$a^2+b^2=c^2$". Mathematical participants, namely the two legs and the hypotenuse, have been encoded as "a and b for legs" and "c for hypotenuse". The mathematical process "sum" has been encoded as "+", "equal" has been encoded as "=", and "square" has been encoded as "^{2}". The mathematical description in mathematical symbolism gives the expression of "Pythagoras' Theorem" in an unambiguous and economical manner. This description in mathematical symbols exactly describes "Pythagoras' Theorem" as it could be applied to other instances on the one hand, the linguistic descriptions of mathematical participants and mathematical processes having been simplified into mathematical symbols through encoding, on the other.

2.16.3 Visual images

In mathematics, visual images are "specialized types of visual representation" (O'Halloran, 2005, p. 15). These types of visual representation are presented in the form of "abstract graphs, statistical graph, diagrams" (O'Halloran, 2005, p. 15) and "geometrical diagrams" (O'Halloran, 2010, p. 218). The function of mathematical visual images is that they "provide a semantic link between the linguistic description of the problem and the symbolic solution" (O'Halloran, 2010, p. 221). This semantic link is the connection between "new sets of grammatical features in mathematical images" (O'Halloran, 2014, p. 8) such as the lines, dots and curves in a geometric image, and the "mathematical entities" (O'Halloran, 2014, p. 8) such as the mathematical participants in language and mathematical symbolism. In Figure 2.11, the explanation of "Pythagoras' Theorem" also involved a visual image of a right-angled triangle.

The geometric image in its own right was labelled with the symbol "┌" and capitalized letters: "A, B, C" to suggest that it is a right-angled triangle with "∠C" as a right angle. Letters "a, b, c" which label the lines suggest that "a" and "b" are two legs while "c" is the hypotenuse in the right-angled triangle.

Viewing the visual image as a complete entity is to see how this visual image as a whole connected with the linguistic description and mathematical symbolism. The visual image is the visualization of the mathematical participants. Different mathematical participants have been visualized in the geometric image. The geometric image in turn offers a convention on which the "physiological perception" (O'Halloran, 2000, p. 363) of mathematical participants could be converted into visual representations. However, the mathematical process such as the "sum", "square", "equal" in linguistic terms, and the "=", "+", "^{2}", have not been explicitly presented in the visual image.

Therefore, the primary function of visual images is its visualization of the mathematical participants. Within the example in Figure 2.11, the operative processes[2] could not be generated.

2.16.4 Connection between different mathematical semiotic resources

The analysis of "mathematical language" must be undertaken "in relation to its co-deployment with mathematical symbolism and visual display" (O'Halloran, 2000, p. 360). The "co-deployment" (O'Halloran, 2000, p. 360) is to take the "contributions and interaction" of mathematical symbolism and visual images into account together with the language. In mathematical language, language as the primary resource is used to establish the mathematical arguments such as the introduction of the mathematical participants and the mathematical process. Mathematical symbolism is used to encode the language into unambiguous and economical symbols that could be adopted into other circumstances. The last component, mathematical visual images, functions to visualize the mathematical participants in both language and mathematical symbolism into the visual representations of graphs, diagrams, figures and the like.

2.16.5 Mathematical discourse: a social-semiotic perspective

Mathematical discourse is constructed from mathematical language. A social-semiotic perspective towards mathematical discourse is that mathematical discourse is understood as the multi-semiotic discourse that involves "language, mathematical symbolism and visual images" (O'Halloran, 2005, p. 11). Semiotic resources within mathematical discourse interact with each other to "construct mathematical reality" (O'Halloran, 2014, p. 9) linguistically, symbolically and visually.

[2] According to O'Halloran (2000), the operative process means: "processes performed on mathematical objects, such as numbers and later, variable and other abstract quantities" (p. 364)

2.17 Traversing the gap between knowledge and representation: a discourse semantic perspective incorporating SFMDA approach

This chapter first reviewed the existing research on how mathematical knowledge is considered in the field of education with reference to different approaches, namely the official approach, sociological approach and social-semiotic approach. Mathematical knowledge is construed as co-related and invisible mathematical concepts. Correlated mathematical concepts indicate that different concepts could be mapped onto a network indicating that they are connected. This network could be interpreted with the help of Bernstein's knowledge structure. Informed by his knowledge structure and the work by Muller, mathematical knowledge structure is interpreted as both horizontally organised and hierarchically organised. In this study, mathematical knowledge is treated as mathematical concept, which is itself "timeless" and "does not exist at any particular time and place" (Mohan, 1986, p.41). Instances of mathematical concepts are "particular" and can be represented by particular semiotic resources.

The knowledge structure that is the internal language of description could be activated into empirical referents. This activation transforms the invisible nature of mathematical concepts into visible representations. Relevant works have mentioned this translation (HKEAA, 2007; Hiebert & Lefevre, 1986) or has theorized this translation (Muller, 2007; Maton, 2011) from a sociological perspective. Drawing from a social-semiotic perspective (Halliday, 1978; Halliday & Matthiessen, 1999), the representation of mathematical knowledge takes the form of a multi-semiotic construct whereby semiotic resources more than language facilitate the construction of mathematical discourses (O'Halloran, 1996, 2005). This social-semiotic perspective on understanding knowledge and representation is favoured in this study.

This chapter then discussed mathematical language and mathematical discourse from two different streams: the perception of the Hong Kong EBD and the social-semiotic approach. Mathematical discourse is constructed from mathematical language in both streams (EDB, 1999; CDC, 2000; HKEAA, 2007; Mok, 2013; O'Halloran, 2000, 2005, 2010, 2014). In the context of practical guidelines, within the EBD's documents, the components within mathematical language have been listed as graphs, figures, symbols, numbers, vocabulary, representations, text and others. The differences and similarities between these components of mathematical language have not been provided. Their connection and interaction have not been defined either.

With the help of the social-semiotic approach of O'Halloran (2000, 2005, 2010 & 2014), the components within mathematical language have been grouped into three categories: language, mathematical symbols and visual images. The connection and interaction between these components is that language foregrounds the mathematical participants and mathematical processes. Mathematical symbolism is used to encode the mathematical participants and mathematical processes with symbols that could be applicable in other circumstances in an unambiguous and economical way. Visual images will visualize the mathematical participants in mathematical images like graphs and figures.

In the next chapter, I will explicate the theoretical foundations with reference to the instances of knowledge representation, and some theoretical considerations behind a social-semiotic approach in the viewing of language.

Chapter Three
Theoretical Foundation:
Recontextualisation in Pedagogic Discourse

3.1 Introduction

This chapter underlines the theoretical foundation of this study. A Systemic Functional Linguistic (SFL) approach to understanding language is associated with Bernstein's (1990) work of pedagogic discourse and recontextualisation. In Section 3.2, I provide a systemic functional model for viewing language and other semiotic systems drawing on the work by Halliday (1978), Halliday and Matthiessen (2004) and Martin and Rose (2003, 2007, 2008). With respect to the subsections of Section 3.2, from Section 3.2.2 to Section 3.2.4, three parameters in viewing language are introduced, namely realization, individuation and instantiation. Following a complementary model between realisation, individuation and instantiation in Section 3.2.5, I begin to concentrate on instantiation from Section 3.2.6. Reinstantiation is introduced as a language phenomenon in Section 3.2.7. In Section 3.2.7, I incorporate a theoretical model, namely the commitment introduced by Martin (2006) that underlines instantiation and reinstantiation. After outlining a trinocular consideration of commitment with respect to ideational meaning commitment, interpersonal meaning commitment and textual meaning commitment in Section 3.2.8, I narrow my focus to ideational meaning commitment through providing two models underpinning ideational meaning commitment. The models of Hood (2008) and Painter and colleagues (Painter et al., 2013) are introduced, with the former concentrating on ideational meaning commitment between different linguistic texts and the latter concentrating on ideational meaning commitment between text and image.

In Section 3.3, I introduce the multi-semiotic nature of mathematical discourse for the purposes of incorporating both recontextualisation and ideational meaning commitment in the interpretation of mathematical discourse. In Section 3.4, I introduce Bernstein's (1990) work on pedagogic discourse and provide a list of different types of pedagogic discourses available in school curricula. In

Section 3.5, I propose the model of curriculum ecology. With the help of this model, the relationship between different pedagogic discourses is connected with each and their connection is understood as a "relay" (p. 2) in Bernstein (1990). In Section 3.6, I suggest that the underlying force that mobilises the relay is recontextualisation. In Section 3.7, I summarize this chapter.

3.2 A systemic-functional model for viewing language and other semiotic resources

In this study, the basic theoretical foundation driving the linguistic analysis is systemic functional linguistics (hereafter SFL) which has been developed in the past six decades. This approach "incorporates the notion that language is a social phenomenon, and in dealing with language it works at the level of the text as a unit of meaning" (Forey, 2002, p. 31). As outlined by Martin (2008), SFL is a theory with multivariate parameters in viewing language where three complementary hierarchies, "individuation, realisation and instantiation" (p. 37) complement each other. This section reviews the three parameters before proposing instantiation as the key parameter adopted for analysis. Although instantiation is the central concern in this study, it is meaningful to discuss the other two parameters, realisation and individuation as well, in order to offer a more comprehensive picture for viewing language.

3.2.1 Realisation

As identified by Martin (2006), most of the work undertaken in SFL has focused on one parameter, realisation (p. 276). The discussion in this section is concerned with realisation.

Systemic functional linguistics is a theory that describes how language makes meaning in social context within a stratified system. Halliday's work (1973, 1978) theorized the relationship between language and context by providing three contextual parameters: field, tenor and mode to capture the ideational meaning, interpersonal meaning and textual meaning emerging from language. The field of discourse indicates "the nature of social activity", the tenor of discourse indicates "the nature of the relationships of participants" and the mode of discourse indicates "the role language play(s) in the situation" (Christie, 2012, pp. 8–9). As for different metafunctions, ideational meaning construes the experience and sequence of the activity, interpersonal meaning enacts the relationships between different participants and textual meaning composes the channel of meaning making.

The relationship between contextual parameters and three metafunctions is developed by Halliday (1978, 1994, Halliday and Matthiessen, 2004). The correlation between three contextual parameters and three metafunctions is provided in Table 3.1.

Table 3.1: Relationship between contextual parameters and metafunctions in a situational context

Register Categories	Realised by	Metafunction
Field: social activity or content	⟶	Ideational (experiential and logical): representing experience and sequence
Tenor: role and status of persons	⟶	Interpersonal: enacting relationships
Mode: role of language in activity	⟶	Textual: channel of meaning making

In Table 3.1, the field of discourse indicates "the nature of social activity", the tenor of discourse indicates "the nature of the relationships of participants" and the mode of discourse indicates "the role language play(s) in the situation" (Christie, 2012, pp. 8–9).

Martin extends this stratified system by providing an extra level of stratification: genre. This extension originates from his revisiting of Halliday's (1973, 1978) work on language and register. Benefited from Halliday's description of register and context, Martin's work offers "a satisfactory account of the goal-oriented beginning-middle-end structure of most texts" (p. 155).

The stratified model in relation to language, register and genre proposed by Martin (2009) is displayed in Figure 3. 1.

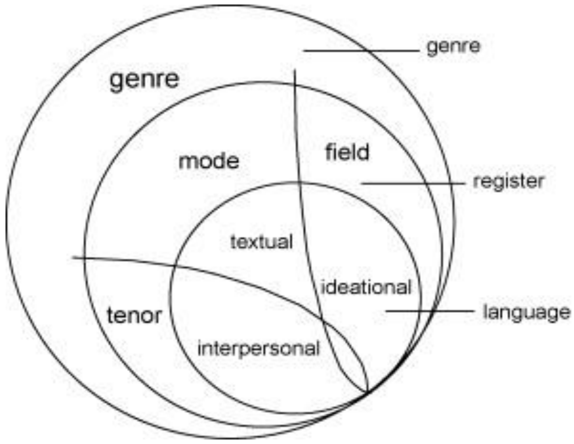


Figure 3.1: Language, register and genre model (Martin, 2009, p. 11)

The relationship between these three circles is that "language functions as the phonology of register, and register and thus language function as the phonology of genre" (Martin, 2010, p. 19). The lexicogrammatical feature of language stands as the inner circle of the model. The middle level is register. The three meta-functions of language include ideational resources, which encode our experiences of the world, interpersonal resources which encode interaction, and textual resources which are concerned with how language is used to organize our experiential and interactional meaning into a coherent whole (Halliday, 1994, p. 35). The notion of register proposed by Halliday and Hasan (1976, p. 22) as "the linguistic features which are typically associated with a configuration of situational features – with particular values of the field, mode and tenor", is composed of three contextual variables. The relationship between context of situation and register variables, as outlined by Christie (2012) is that "the meanings of any context of situation will depend on the ideational meaning (expressing the field), the interpersonal meanings (expressing the tenor and relationships of participants), and mode (the manner of organizing the text as a message)" (p. 9). The overarching layer of that model is genre. A working definition of genre is that genre is "a staged, goal oriented, social process" (Martin and Rose, 2008, p. 6). – staged as there may be a number of steps in any one text; goal oriented as each text is constructed with a goal in mind; and social because each text involves writer and readers and/or speakers and listeners (Martin and Rose, 2008). This working definition has been widely applied as the theoretical framework to analyse the language of education, generating a linguistic school known as the "Sydney School" in Australia since the 1980's. A number of research projects undertaken in Sydney and elsewhere since the 1980's focuses on the investigation of key genres within education (e.g. Martin, 1992; Matthiessen et al., 1992; Christie and Martin, 1997; Martin & Veel, 1998; Macken-Horarik, 2002 & McCarthy et al., 2002; Coffin, 2004 & 2006; Gibbons, 2006; Firkins et al., 2007; Martin & Rose, 2007; Martin & Rose, 2008; Coffin and Derewianka, 2008;). Australia has led the way by introducing this genre-based pedagogy into the school system including composing the curriculum document and guiding teaching and learning (Rothery 1996; Christie & Martin 1997; Feez, 1998; Macken-Horarik 2002; Martin and Rose 2008; Rose and Martin, 2012). This approach is now applied in wider contexts around the world (e.g. Schleppegrell, 2004), and adapted to second/foreign language education (e.g. Schleppegrell & Colombi, 2002; Byrnes, 2006; Rinner & Weigert, 2006). Later, this genre-based pedagogy was formulated by Christie (2005) and allowed teachers to help students to understand and recognize key sets of textual and linguistic features within different discourses of the curriculum in their writings. The findings from such studies have been adopted, forming the foundation of the genre-based approach to teaching.

Evolving from these studies, genre-based pedagogy (Rothery, 1986; Rose and Martin 2012; Christie, 2012) has been conducted to extend and investigate the discursive pedagogic practices in education. Apart from its special focus on classroom discourses and students' writing, interest in how various curriculum documents are composed and how these curriculum documents are practiced in various setting of pedagogy are now emerging. For example, how textbooks facilitate the knowledge-building processes of students have attracted much attention. These include several ongoing research projects undertaken within the Department of English, The Hong Kong Polytechnic University either fully or partially selecting textbooks as their source of data. These include but are not limited to a History textbook by Akashi (forthcoming), Science textbooks by Forey and her colleagues (2012), English textbooks by Guo as her PhD research (Guo, 2015). Studies adopting realisation as the parameter of analysis, treat texts as the unit of analysis. Meaning unfolds as a text, moving from the smallest unit of phonology and graphology to the end of the genre. This approach tracks the ontological development of the text and addresses how individual text makes meaning ontologically.

Much work in the SFL tradition has taken realisation as the parameter of investigation, since the process of realisation entails "the move from meaning to form" (Kress, 2007, p. 17). However "much less work has concerned itself with two complementary parameters: instantiation and individuation" (Martin, 2006, p. 276) which have different orientations to the focus of realisation.

3.2.2 Individuation

The second parameter is individuation. In order to explore individuation, according to Bernstein's (2000) the distinction between *repertoire* and *reservoir* (p. 158) needs to be explained. As acknowledged by Martin (2008, p. 35), Bernstein's (2000) distinction between *repertoire* and *reservoir* (p. 158) has been accounted for by the relationship between individual and society. This relationship is modelled with the assistance of individuation. Bernstein (2000) states that:

> Here a distinction can be made between the set of strategies any one individual possesses and their analogic potential for contextual transfer, and the total sets of strategies and their analogic potential possessed by any one individual and the term reservoir to refer to the total of sets and its potential of the community as a whole. Thus, the repertoire of each member of the community will have both a common nucleus but there will be differences between the repertories. There will be differences between the repertories because of differences between the members arising out of differences in members' contexts and activities and their associated issues. (p. 158)

By deduction from the foregoing quotation, there is a part-whole relationship between *repertoire* and *reservoir,* with the former standing for "each member of the community" (Bernstein, 2000, p. 158) and the latter standing for "the total of sets and its potential of the community as a whole" (Bernstein, 2000, p. 158).

Individuation is a process whereby the *reservoir* of the whole community is narrowed to the *repertoire* of an individual, or in Bernstein's (2000) terms, "the flow of procedures from *reservoir* to *repertoire*" (p. 158). Figure 3.2 is a model of individuation specifying the flow from *reservoir* to *repertoire*: from the community to the individual member.

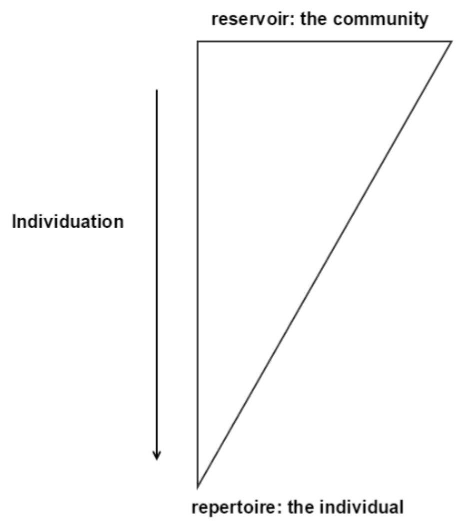


Figure 3.2: Model of individuation: the flow from *reservoir* to *repertoire*

Individuation indicates the process whereby an individual member privatises the community's practices such as identity, knowledge and pedagogy as the *reservoir* into the individual's own *repertoire*. In the SFL tradition, individuation could work with how the identity of the society has been legitimated in individual members, such as the building of an individual's identity in contrast to the society's ideology (Martin, 2006, p. 294). With respect to the dissemination of knowledge in education, individuation could account for how knowledge has been privatised in different members from the same education community. For example, individuation could help to describe the disparity in knowledge acquirement between different students in a classroom through working out how each student performs differently acquiring the same knowledge. With respect to mathematics, individuation will be particularly useful in understanding studies working on how the same mathematical concept has been understood by different students through working on the classroom discourses such as teacher-student interactions, students' oral and written feedbacks and the in-class and off-class assignments. Although not explored in this present study, repertoire could describe how individual students perceive Pythagoras' Theorem while reservoir accounts for the knowledge and pedagogy used for teaching Pythagoras' Theorem by schools.

3.2.3 Instantiation: from system to instance

The third parameter is instantiation, first developed by Halliday (1991, 2007) in describing the relationship between "system" and "instance". Drawing on Halliday' work, Martin (2008) defines instantiation as "a scale of generalization, aggregating the meaning potential of a culture across instances of use" (p. 33). Instantiation "interprets the relation of system to instance" (Martin, 2006, p. 276). With reference to the relationship between instance and system, Hood (2008) suggests that instantiation is a "relationship of a single text to the whole system of language as available potential" (p. 352), highlighting a single-whole analogy. This single-whole analogy could be better understood with the assistance of the analogy between climate and weather exemplified by Halliday (1991). The relationship between instance and system resembles the analogy between climate and weather:

> Climate and weather are not two different things; they are the same thing that we call weather when we are looking at it close up, and climate when we are looking at it from a distance. The weather goes on around us all the time; it is the actual instances of temperature and precipitation and air movement that you can see and hear and feel. The climate is the potential that lies behind all these things; it is the weather seen from a distance by an observer standing some way off in time. So of course there is a continuum from one to the other; there is no way of deciding when a "long term weather pattern" becomes a "temporary condition of climate", or when "climatic variation" becomes merely changes in the "weather."(p. 9)

In Halliday's analogy, climate is the system and weather is an instance within the system of climate. For the system of climate, different weather phenomena such as cloudy, rainy and sunny could be mapped onto that system of climate. Each weather phenomenon stands for its own characteristics and meanwhile contributes to the complete system of climate. As articulated by Halliday (2003), new instance provides "a new interface, another kind of instantiation through which changes in the system could take place" (p. 131). Halliday (2004) also suggests new instance for the same system "will not change the principles of theory", but "it will add a new type of instantiation" (Halliday, 2004, p. 220)." This analogy could be used to describe the relationship between mathematical concept and different instances of the same mathematical concept where mathematical concept as the principle of theory remains unchanged and new instances "will broaden our conception of possible kinds of reality" (Halliday, 2004, p. 220) through the addition of new types of instantiation.

Martin and White (2005) theorize the climate-weather analogy through providing a cline of instantiation to account for this analogy.

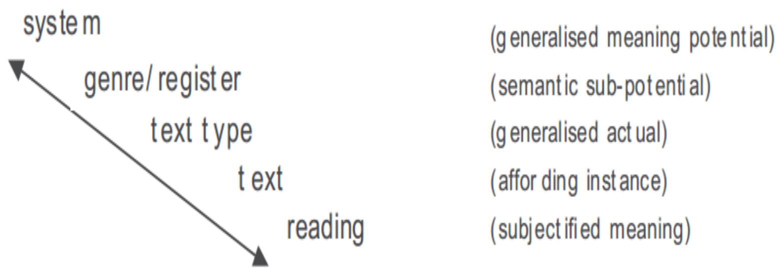


Figure 3.3: Model of instantiation (adapted from Martin & White, 2005, p. 162)

The model in Figure 3.3 is a model of a cline of instantiation, which represents the parameter of instantiation. This model "leads us to look at linguistic phenomena variably from the perspective of language as meaning making potential and from the perspective of the instantiation of that potential in individual texts" (Martin and White, 2005, p. 162). Moving down the scale from system, "the meaning potential of the language as a whole becomes progressively narrowed" (Martin, 2006, p. 285), first to "its generic and registerial sub-potentials" (Tang, 2013, p. 24). Moving further is text type, representing "generalized instances of characteristic texts" (Tang, 2013, p. 24) followed by text as "the actual instance of an individual text" (Tang, 2013, p. 24) which stands as "an instance of language" (Martin, 2006, p. 285). The lowest level of instantiation in this model is labelled "reading" which refers to "the meaning taken from the text according to the subjectivity of the reader" (Martin and White, 2005, p. 162). Because "reading" is subjectively decided by the reader, it means that "any text is itself a meaning potential" (Hood, 2008, p. 354).

Instantiation suggests that the overall meaning potential of a system is accumulated based on different instances, therefore, "any single instance of text is then an actualization of meaning in relation to a generalised system of language" (Hood, 2008, p. 353). As one of the three dimensions, instantiation which interprets the relationship between system and instance, becomes "a valuable theoretical tool in a study of how meanings change as one text is reconstructed from another" (Hood, 2008, p. 353). Alternatively, we can interpret the other way round. The overall meaning potential is compartmentalised as different instances, with each instance carrying its own meaning on the one hand and being determined by the overall system on the other hand.

According to Halliday and Matthiessen (2004), "the system of a language is 'instantiated' in the form of text" (p. 26). Language is not "the sum of all possible texts", however language is "a theoretical entity to which we can assign certain properties and which we can invest with considerable explanatory power" (Halliday & Matthiessen, 2004, p. 27). In terms of the mathematics examined in the presented study, the collection of pedagogic texts concerned with one mathematical concept is not just the sum of all possible texts in the education system with respect to this mathematical concept. This collection has an explanatory power to answer how the system has been instantiated as different instances regarding this mathematical concept. The relationship between system and language is "analogous to that between weather and the climate" (Halliday & Matthiessen, 2004, p. 26). Similarly, there is an analogous relationship between a complete collection of texts and one specific instance of that mathematical knowledge. Generally, this relationship could be seen as the relationship between a system of representations of a specific mathematical concept and one specific instance of the representations at the semiotic level because "these patterns of instantiation show up quantitatively as adjustments in the systemic probabilities of a language" (Halliday & Matthiessen, 2004, p. 27). Hence, according to Painter et al. (2013) "instantiation is the relation between the meaning potentials as a whole and the particular selections and realisations from that system that are actualised in an individual text" (p. 134), or in other words, instantiation accounts for the relationship between "the potentiality for being instantiated" (Halliday, 2003, p. 257) and what has been instantiated. According to Halliday (2007), the whole is the "potential", which is "a resource that you draw on in reading and writing and speaking and listening-and a resource that you use for learning with" (p. 274) while the particular is the "text" which refers to "all the instances of language that you listen to and read. And that you produce yourself in speaking and writing" (p. 274). Following the elaboration by Painter et al. (2013), the parameter of instantiation offers an exploratory power to account for the relationship between the whole and the particular.

3.2.4 Complementarity of realisation, individuation and instantiation

Martin (2008) proposed a provisional model for the three complementary dimensions identified in SFL.

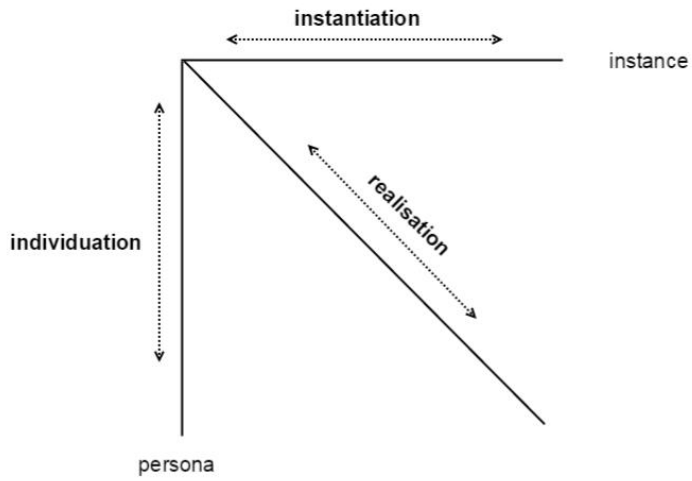


Figure 3.4: Three complementary parameters (adapted from Martin, 2008, p. 37)

As outlined in Figure 3.4, each parameter highlights its specific focus when exploring the system of meaning making. As noted by Martin (2008) "realisation, instantiation and individuation" as three parameters must be kept in mind "when exploring semantic variation, since all systems proposed for a given language and culture along the realisation hierarchy instantiate, and all individuate as well" (p. 57). These three parameters complement each other with individuation addressing the relationship between individual and the community, instantiation addressing the relationship between instance and system and realisation addressing the strata organisation of meaning.

Since the present study focuses on understanding how mathematical knowledge has been presented as different instances between a series of correlated pedagogic discourses, the work of instantiation is selected as the underlying parameter. This selection is because, according to Hood (2008), "any single instance of text is then an actualization of meaning in relation to a generalised system of language" (p. 353). In this study, any single instance of text regarding one mathematical concept is one of "actualization of meaning" (Hood, 2008, p. 353) in relation to "a generalised system" (Hood 2008, p. 353) regarding this mathematical concept. To interpret from another perspective, this generalised system is segmented into different instances of the same mathematical concept, with each instance having its own way of meaning making.

Following the track of instantiation, two relevant theoretical considerations, reinstantiation and commitment, will be considered to understand how different instances concerned with the same mathematical concept are differentiated. Reinstantiation deals with different instances in relation to the same mathematical concept; meanwhile commitment is selected as the model to explore the parameter of instantiation. Commitment will account for the amount of meaning instantiated in one instance. Section 3.2.6 will explore reinstantiation. Section 3.2.7 will discuss different types of commitment, namely, ideational meaning commitment, interpersonal meaning commitment and textual meaning commitment before focusing on ideational meaning commitment in Section 3.2.8.

3.2.5 Reinstantiation and serial reinstantiation

This study is theoretically underpinned as an SFL-informed research (Halliday & Matthiessen, 2004). Instantiation is selected as the parameter for analysis since Halliday and Matthiessen (2004) state that "the system of a language is 'instantiated' in the form of text" (p. 26). Language is not "the sum of all possible texts", however language is "a theoretical entity to which we can assign certain properties and which we can invest with considerable explanatory power" (Halliday & Matthiessen, 2004, p. 27). Likewise, the collection of the pedagogic texts concerned with one mathematical concept is not just the sum of all possible texts in the education system. This collection provides an explanatory power to answer how the system has been instantiated as different instances regarding this mathematical concept. As noted previously, the relationship between system and language is "analogous to that between weather and the climate" (Halliday & Matthiessen, 2004, p. 26). Generally, this relationship could be seen as the analogy between the potentiality – meaning potential as a whole – and the probabilities – collections of particular selections – because "these patterns of instantiation show up quantitatively as adjustments in the systemic probabilities of a language" (Halliday & Matthiessen, 2004, p. 27). Hence, according to Painter et al. (2013) "instantiation is the relation between the meaning potentials as a whole and the particular selections and realisations from that system that are actualised in an individual text" (p. 134). It must be noted that "realisations" offered in the work by Painter and her colleagues (2013) are not treated as one of the parameters of language. Rather, realisations used here refer to texts in general sense. In the present study, one mathematical concept could be viewed as "the meaning potentials as a whole" while different instances of that mathematical concept could be viewed as different "particular selections and realisations" (Painter et al., 2013, p. 134) or as different "texts" in Halliday's (2007, p. 274) sense.

With the help of the prefix re-, (re-)instantiation and its verbal form (re-)instantiate both denote the process: "to instantiate again" (Hood, 2008, p. 356), signifying a process of change. "How meanings change in the process of reinstantiation" (Hood, 2008, p. 356) is central in understanding the relationship between different pedagogic texts. To be more explicit, this central question is to ask "how does the meaning potential of one differ from the meaning potential of the other?" (Hood, 2008, p. 356). The change between different texts represents "the serial re-instantiation" (Hood, 2008, p. 352) from one text to another.

In this study, the change between different pedagogic texts is viewed as a logogenetic evolution. A logo-genetic evolution is originally associated with "the development of a single text" (Hood, 2008, p. 351) through understanding how each text unfolds, "from a beginning through a middle to an end" (Halliday &

Martin, 1993, p. 18). This logo-genetic evolution expands from understanding the change within one text to the understanding of the change between different texts within a curriculum. This logo-genetic evolution between different pedagogic texts within a curriculum is termed curriculum ecology, as explored in Section 3.5

Re-instantiation is a theoretical framework that is proposed to underline "what is happening in the instance, and at the same time an enrichment of the theory to account for different contexts of use" (Hood, 2008, p. 353). In the present study, re-instantiation accounts for how one mathematical concept has been instantiated differently in different educational contexts.

3.2.6 Re-instantiation and commitment

Studies that focus on instantiation (Martin, 2006; Hood, 2008; Caple, 2009; Chang, 2011; Zhao, 2012; Painter, et. al, 2013) have proposed "commitment" as the theoretical model that underlines "degree of meaning potential instantiated in one instance or another" (Hood, 2008, p. 356). In this section, how "commitment" accounts for the instantiation of meaning potential in multi-semiotically-constructed instances is revisited with a focus on ideational meaning in particular. Movements in the commitment of ideational meaning are theorized for the purposes of capturing the "serial re-instantiation" (Hood, 2008, p. 356) between different pedagogic texts.

Situated in the work on instantiation by Halliday (Halliday, 1991; Halliday & Matthiessen, 2004), recent developments in the work on re-instantiation have been formulated by Martin and colleagues (e.g. Martin, 2006, 2008; Hood, 2008; Caple, 2009; Painter et al., 2013). Commitment is a theoretical model describing instantiation. This theoretical model contributes to discussing "the amount of meaning potential that is taken up from any particular meaning system in the process of instantiation" (Painter et al., 2013, pp. 148–149). Studies on re-instantiation and commitment can be categorized into two types with reference to the physical constraints of research data. They could depict the relationship between different texts that appear in different documents such as the work by Martin (2006) in his analysis of four related war stories published in four different sources. They could also depict the relationship between different texts that are framed within one physical constraint (e.g. one page), such as the work by Hood (2008) contributing to the discussion between source text, summary and notes which have been grouped onto one page and Bateman's (2008) work on how information is divided on one website page.

In terms of the semiotic systems involved in re-instantiation and commitment, the work in this field derives from the analysis between different linguistic texts (Martin, 2007; Hood, 2008) and later develops to include other semiotic systems such as visual images in children's picture books (Painter, et al., 2013), and in forms and diagrams in physics textbooks (Zhao, 2012). The development of the work on re-instantiation and commitment paves the way forward for a wider-coverage of research domains that these theoretical underpinnings could include.

In this present study, re-instantiation has been extended to cover serial changes between different texts that are not restricted to one document. The extension of re-instantiation is based on the types of pedagogic documents to be explored in this present study. Commitment as the theoretical model underlying re-instantiation has been extended to cover semiotic resources more than linguistic resources. The extension of commitment concerned in this present study is based on the nature of mathematical discourse. Section 3.7 will discuss the nature of mathematical discourse. Section 3.8 will propose a curriculum ecology on which the change between different pedagogic texts within a curriculum could be examined.

3.2.7 Commitment: ideational meaning, interpersonal meaning and textual meaning

In this section, I am concerned with the nature of commitment. Language in SFL has three functions: to construe ideational meaning, to enact interpersonal meaning and to compose textual meaning. Following this tradition, the delicate work on commitment considers instantiation from a trinocular perspective as well, involving the consideration of ideational meaning, interpersonal meaning and textual meaning. In order to reveal comprehensively how the meaning potential has been instantiated in one instance, each of the three perspectives must be taken into consideration. This consideration corresponds with the discourse semantic (Martin & Rose, 2003, 2007, 2014) perspective in theorizing the phenomenon of language. Since the central concern of re-instantiation is about "change" (Martin, 2006; Hood, 2008), how language changes from one instance to another, the change that this study is concerned with takes place across these three meta-functions. Commitment therefore has been proposed to measure the degree of meaning potential, including three different perspectives: ideational, interpersonal and textual. To consider the commitment of meaning potential by one instance of language and to argue how its level of commitment differs from other instances is actually a threefold consideration. In terms of ideational meaning, the level of commitment should measure the inscription of the taxonomic relationship and sequential relationship both embedded within lexical items such as the nominal groups and verbal groups. In terms of interpersonal meaning, the level of commitment should measure the enactment of appraisal system with respect to the different degrees of evaluation expressed in different instances. Textually speaking, the level of commitment should measure the flow of information in terms of how the periodic system is established. All the above three systems instantiate the meaning potential through its own specific ways in the construing of ideational meaning, enacting interpersonal meaning and composing textual meaning. Each instance of language has the potential to be viewed from a trinocular perspective. This trinocular perspective offered by instantiation confirms Halliday's consideration that "each act of meaning instantiates numerous underlying systems" (Halliday, 2006, p. 35). Meaning potentials encompass ideational meaning, interpersonal meaning and textual meaning. Each of the above three meaning potentials has its own specific means of commitment, addressing different instantiation processes.

3.2.8 Commitment of ideational meaning

In this section, the commitment of ideational meaning is proposed as the model to underline the phenomena of re-instantiation identified in this study. As has been outlined in Sections 3.2.5 and 3.2.6, re-instantiation and its analytical model "commitment" are proposed to account for how meaning potentials have been committed in instances of language. The central focus of this study departs from an ideational perspective because, as considered by Martin (2007), “in the stratified model of context, the register variable field provides a social semiotic perspective on knowledge structure” (2007, p. 34). That is to say, from a linguistic perspective, “knowledge is by and large realised through, construed by, and over time reconstrued through ideational meaning” (Martin, 2007, p. 34). Therefore, the exploration of knowledge structure from a linguistic perspective entails the “exploration of field of discourse” (Martin, 2007, p. 34). Following Martin’s (2007) explanation, the commitment of ideational meaning in different instances is explored in this present study.

Since the exploration of knowledge structure has been “treated linguistically as exploration of field of discourse” (Martin, 2007, p. 34), two compulsory steps in viewing ideational meaning commitment will be conducted in this study. The first compulsory step is to investigate what ideational meaning has been committed in each instance. With the help of this step, the commitment of ideational meaning for each instance can be identified. The second step is to work out the relationship between different instances through comparing their differences in terms of the commitment of ideational meaning, i.e. their different commitments of participants, process and circumstances.

With respect to the exploration of ideational meaning commitment, Hood (2008, pp. 357–361) offered five types of relationships. They are generalisation, abstraction, grammatical metaphor, lexical metaphor and infusion. These five types of relationship have been proposed to account for the commitment of ideational meaning in different instances through working out the “specific changes in wordings/meaning” (Hood, pp. 357–361) by illustrating and comparing their experiential features such as participants, processes and circumstances in her data.

In terms of generalisation, Halliday and Matthiessen (1999) describe it as a lexical relationship that allows for “the development of extended taxonomic hierarchies” (p. 615). Hood (2008) suggests that generalisation relates “individual entities to general classes of entities” (p. 357). Generalisation relates to “classificatory relationships of hyponomy (kinds of) and compositional relationships of meronomy (parts of)" (Hood, 2008, p. 357), informed by the work of Martin and Rose (2003, 2007, 2014) on lexical relationship. Generalisation incorporates this linguistic approach with knowledge construction by Bernstein (2000) in the sociological approach. Drawing from Bernstein's (2000) knowledge underlined in Chapter Two, the relationship between "individual entities" (Hood, 2008, p. 357) to "general classes of entities" (Hood, 2008, p. 357) is formed as a relationship of the hierarchical knowledge structure. Meanwhile the relationship between different "individual entities" (Hood, 2008, p. 357) within a "general class of entities" (Hood, 2008, p. 357) is formed as a relationship of horizontal knowledge structure. From a retrospective standpoint, hyponymic relationship and meronymic relationship provides two further sub-classes of relationship within the hierarchical knowledge structure, classifying the forms of absorption as either "part of" or "kind of".

In terms of abstraction, according to Hood (2008), "abstraction has to do with the reconstrual of experience from an everyday commonsense representation of the world to some kind of decontextualised representation" (p. 358). Linguistically speaking, abstraction derives from Halliday and Matthiessen's (1999) work on abstract words, which are "construing some aspect of our experience, but there is no concrete thing or process with which then can be identified" (p. 617). In this sense, abstraction and abstract words are "decontextualised representation" (Hood, 2008, p. 358) compared with "everyday commonsense representation of the world" (Hood, 2008, p. 358). The abstraction is in the nature that concreteness in terms of the thing or process is unidentifiable. This linguistic exploration can be related to Bernstein's (2000) sociological account of knowledge structure, and Maton's (2013) work on Semantic Gravity, since both depict the movement between abstractness and concreteness based on the contextual dependency level. Working towards abstraction will be useful in underlining the relationship between different instances of knowledge in the way of capturing their different level of commitment in terms of their instantiation of knowledge. The relationship between abstract knowledge and concrete examples in the sense that knowledge displayed as technical term is more decontextualised than concrete examples. For example, with reference to Figure

2.2, the embedding of Pythagoras' Theorem as a technical term is more abstract than the embedding of Pythagoras' Theorem as the Ting Kau Bridge since a real-life context: the bridge is included.

In the terms of grammatical metaphor, it is about "a lexico-grammatical choice of a process meaning being reconstrued as a Thing" (Hood, 2008, p. 350). Grammatical metaphor which prevails in scientific discourse (Halliday & Martin, 1993), could be revisited from a re-instantiation and commitment process in looking at how ideational meaning has been committed differently when "we re-instantiate meanings from one text into another" (Hood, 2008, p. 360).

In the terms of lexical metaphor, it draws from the classical works on metaphor study (cf. Lakoff & Johnson, 1988), in understanding the meaning embodied between metaphoric expression and congruent expression, by predicting how the lexical items available in the texts have pre-stored some metaphorical meaning which, based on the literally congruent expression, could not be fully appreciated. Drawing from Bernstein's (1999) work on recontextualisation, we can argue that metaphorical meaning has already been delocated from its "source domain" and relocated into the "target domain" which is the metaphorical expression to be encountered in the text.

In terms of infusion, the change between processes may also result in the infusion of other experiential categories (participants and/or circumstances). For example, in Hood's (2008) work, the change from "consider" to "reassess" infuses "additional circumstantial meaning" (p. 360), because, "reassess" means "consider again and evaluatively". Of course, we can categorise the relationship between "consider" and "reassess" as a type of generalisation because "reassess" is a kind of "consider", forming up a compositional relationship. However, to capture their relationship more precisely and to indicate that "reassess" commits more ideational relationship in terms of the circumstantial features of "frequency" and "manner", infusion is proposed because "the relationships that adhere are of specifying more or less circumstantial meaning potential through infusion/defusion of lexical verbs" (Hood, 2008, p. 360).

Table 3.2 synthesizes Hood's (2008) work on how ideational meaning is committed and modelled in language.

Table 3.2: Hood's (2008) work on ideational meaning commitment

Relationship	**Description**	**Categories**	**Examples(drawn from Hood, 2008)**	
Generalisation	Individual entities to general classes of entities	Kinds of: classificatory relationships of hyponomy	For example: Librarian is a kind of job.	
		Parts of: compositional relationships of meronomy	For example: Arm is a part of the human body	
Abstraction	The reconstrual of experience from an everyday common-sense representation of the world to some kind of decontextualised representation	The movement between abstractness and concreteness based on the contextual dependency level	For example, the relationship between abstract knowledge and concrete examples in the sense that knowledge displayed as technical term is more decontextualised than concrete examples.	
Grammatical metaphor	The reconstrual of process into a Thing	Process to Thing	fail (verb)	failure(noun)
Lexical metaphor	The recontextualisation between "source domain" and "target domain"	The transition between metaphoric expression and congruent expression	Congruent expression: *change* commits less circumstan–tial meaning potential.	Metaphoric expression: *make a break* commits more circumstan–tial meaning potential.
Infusion	The change between processes may also result in the infusion of other experiential categories (participants and/or circumstances).	Process = Process + participant and/or circumstances	For example, the change from "consider" to "reassess" infuses "additional circumstantial meaning" (p. 360), because, "reassess" means "consider again and evaluatively".	

As shown in Table 3.3, Hood's work (2008) has concentrated on the complementary relationship between linguistic resources when the instances are language in general sense. Recent developments in multi-semiotic studies (e.g. Painter et al., 2013) have developed a complementary relationship between language and visual image in terms of the commitment of ideational meaning in both verbiage and image. Table 3.3 is adapted from Painter et al. (2013, p. 138), focusing on the complementary ideational meaning systems across image and language.

Table 3.3: Complementary ideational meaning systems across image and language (adapted from Painter et al., 2013, p. 138)

	Visual meaning potential	Visual realisations	Verbal meaning potential	Verbal realisations
Action	Visual 'action'	Depicted action with:	Action figures	Tense, phase, etc with transitivity structures
	Action	Vectors		Material, behavioural processes
	Perception	gaze vectors		Mental perception processes
	Cognition	Thought bubbles, face/hand gestures		Mental cognition processes
	Talking	Speech bubbles, face/hand gestures		Verbal, behavioural processes
	Inter-event relations	Juxtaposition of images(+/- change of setting or participant)	Conjunction, projection	Logicosemantic relations of expansion and quoting/reportin g
Character	Character attribution	Depiction of physical attributes	Participant description, classification,	Relational transitivity nominal group
	Character manifestation and appearance	Character depiction	identification	structures, deixis
	Character relations	Adjacent/symmetrica l arrangement of different participants	Participant classification, description	Comparative epithets; classifying clauses, etc.
Setting	Circumstantiatio n	Depiction of place, time, manner	Circumstantiatio n	Specification of time, place, cause, manner, matter, contingency, role, etc.
	Inter-circumstance	Shifts, contrasts continuities in locations		Logicosemantic relations of enhancement

Table 3.3 is interpreted from an experiential perspective. Action is associated with Process, Character is associated with Participant, and Setting is associated with Circumstances. Both language and visual image contain the potential to commit all the three experiential potentials.

The central concern of the Table 3.3 is to map out the possible ways in underlining bimodal texts when the commitment of ideational meaning is achieved in both verbal language and visual images. The complexity of a bimodal text (verbal-visual text) derives from the fact that "there is more than one meaning system at play" (Painter et al., 2013, p. 134). In order to understand such as text, "two complementary sets of meaning systems" must be mapped out (Painter et al., 2013, p. 134). We need to "track the way each is instantiated in the text so as to compare their relative contributions to overall meaning" (Painter et al., 2013, p. 134). In instances where neither the visual nor the verbal mode on its own "carries the full meaning nor provides the full enjoyment and fun of the text", it requires "negotiating the gap between the two modalities and thus arriving at a new meaning that results from the co-patterning of semiotic resources" (Painter et al., 2013, p. 148).

The incorporation of the works illustrated above invites this study to think of how the re-instantiation of knowledge and the different levels of ideational meaning commitment deal with the research data selected in this study. The semiotic nature of mathematical discourse which has been discussed in previous chapters will be revisited here to underpin the ideational (experiential in particular) construction in mathematical discourse.

3.3 The semiotic nature of mathematical discourse

With respect to the present study, the multi-semiotic nature of mathematical discourse informs two things. First, the relationship between verbal language and mathematical symbolism can be treated as a type of recontextualization. For example, the meaning of "percentage" are encoded as "%" in symbolic form. Second, mathematical visual image is treated as a type of image that could be investigated from an instantiation perspective through using the work of Painter and her colleagues (Painter et al., 2013) on the ideational meaning commitment between linguistic text and visual image. In this study, the instances for achieving ideational meaning commitment are extended to cover different types of representations in mathematics, such as table taxonomy, flowchart, technical term, symbolic equations, linguistic descriptions and visual images.

It has been well acknowledged that multimodal features are predominantly identified in mathematical discourse (e.g. Lemke, 1998; O'Halloran, 2000, 2003, 2005, 2007a). The use of semiotic resources such as: mathematical symbolism and visual images alongside verbal language results in extensive discussion on considering mathematical discourse as an artefact whose meaning-making processes are discursive and multi-layered (O'Halloran, 2007b). Drawing on Halliday and Matthiessen's (2004) systemic functional linguistics (SFL) and Martin and Rose's (2007) discourse semantics, O'Halloran (2005, 2007a, 2007b) considers mathematical discourse from the social-semiotic perspective, arguing that the meaning-making process in mathematics involves the complex and constant interplay between three mathematical resources: verbal language, mathematical symbolism and visual images.

Recontextualisation is identified in mathematical discourse. For example, mathematical symbolism adopts the experiential potential of verbal language, functioning as participants, processes and circumstances that were originally the role played by verbal language. This recontextualisation between verbal language and mathematical symbolism indicates that "language functions as the meta-discourse" (O'Halloran, 2005, p. 75) for mathematical symbolism which is incorporated with "the nature of experiential meaning in mathematics" (O'Halloran, 2005, p. 75).

The encoding of experiential meaning into mathematical symbolism allows the organisation of mathematical discourse in an economic manner. Economic is interpreted in the sense of both the textually economic and the spatially economic. The textually economic is concerned with the use of less wording and the spatially economic is based on the reality that mathematical symbolism occupies less space compared with its lexicalized version. For example, if we compare “percentage” with “%”, we can have a perception that although both the terms express the same meaning, “%” only occupies one character position while “percentage” occupies 10 position since it contains 10 letters. The recontextualisation relationship between mathematical symbolism and verbal language is termed semiotic adoption in O’Halloran’s (2007b) work. Encoding the experiential potential of verbal language, mathematical symbolism has positioned itself as a part of verbal language, mixed with verbal language in compiling mathematical discourse.

Mathematical symbolism construes the experiential potential of verbal language, standing together with verbal language to indicate that one pedagogic item is complicated in nature. Of course, we can argue that one pedagogic item is a bimodal artefact, composed of verbal language and mathematical symbolism as its two semiotic systems. Through rendering the symbolic meaning into the verbal explanation, we can have a linguistically rendered version with one symbolism being rendered into one or a cluster of nominal groups. This rendering of meaning reveals that the encoding process is performed by mathematical symbolism. To argue one pedagogic item as a whole, this rendering of meaning could tell how the experiential meaning potential has been encapsulated in the mathematical symbolism through comparing with the rendered linguistically version.

Rather than draw an explicit line between different semiotic resources in mathematical discourse, (such as bimodal or trimodal), this study prefers to term the phenomenon as semiotic complex. In O’Halloran’s (2007b) terms, this semiotic complex will be a semiotic adoption and semiotic mixing; semiotic adoption in the sense that mathematical symbolism adopted the experiential potential of verbal language and semiotic mixing in the sense that each pedagogic item is a complexity of different semiotic resources within which different semiotic systems mixed with each other.

Visual images in mathematical discourse, can be further divided into different categories:

- geometric image for the purpose of illustration (e.g. to illustrate the experiential features such as participants, process and circumstance in visual representation);
- mathematical matrix table for the purpose of calculation (e.g. in calculus, the matrix table indicates the experiential features of process, multiply in particular);
- relationship table for the purpose of illustrating the textual relationship,
- diagram for the purpose of illustrating the experiential feature of lexical relationship (knowledge relationship),
- coordinate for the purpose of illustrating experiential potentials (the participants, the process and the circumstance).

Therefore, to identify the experiential features in different instances of knowledge representation is crucial in underpinning the differences of commitment of ideational meaning. As for verbal language, the commitment of ideational meaning could be revealed through the work of comparing the changes of wording and in turn the changes of meanings. When multi-semiotic phenomena occur, the first consideration is to think of whether the instance is a semiotic complex where mathematical symbolism encodes the experiential potential of verbal language. If the phenomena also include a visual image, one key concern is to identify what experiential features in the semiotic complex have been preserved in the mathematical visual image and to what degree the experiential meaning is preserved. Speaking of the knowledge construction, the work on commitment will reveal in great detail how the knowledge structure of mathematical knowledge (in this study, the Pythagoras' Theorem) has been instantiated in different pedagogic items, and to what degree the semiotic resources involved have contributed to the knowledge representation as scaffold by different semiotic constructions and semiotic combinations in mathematical discourse.

3.4 Interpreting pedagogic discourse and providing a working definition

This study derives from my personal interest in understanding how different texts in one curriculum scaffolds the representation and progression of knowledge across that curriculum. For the purpose of conducting educational research from a discourse analytical perspective, I will first determine the scope and criteria for unit of analysis. Bernstein's model of pedagogic discourse (1990, 2000) is revisited here to lay down the fundamental properties about pedagogic texts, which are the central focus in the present study. Bernstein (1990) defines pedagogic discourse as

> "the rule which embeds a discourse of competence (skills of various kinds) into a discourse of social order in such a way that the latter always dominates the former. We shall call the discourse transmitting specialized competences and their relation to each other instructional discourse, and the discourse creating specialized order, relation, and identity regulative discourse." (p. 183).

In his definition, pedagogic discourse is "a principle for appropriating other discourses and bringing them into a special relation with each other for the purposes of their selective transmission and acquisition" (Bernstein, 1990, p. 159). Hence, pedagogic discourse "is a discourse without a specific discourse" (Bernstein, 1990, p. 159). Bernstein's clarification of discourse might be contradictory to our linguistic interpretations of this term. In fact, what Bernstein tries to outline is that pedagogic discourse is a set of rules. These rules regulate the construction of specific discourses. Christie (1995) offered a linguistic exploration towards Bernstein's pedagogic discourse in terms of Halliday's (1994) theory of register. In her work, Christie (1995) suggests that Bernstein's pedagogic discourse, which is a set of rules, resembles the functional notion of register because its relation "to Bernstein's meaning is very close" (p. 224). The closeness between *discourse* in Bernstein's terms and *register* in Halliday's functional grammar is because the two subsets of pedagogic discourse, regulative discourse and instructional discourse, are both describing how texts are conditioned in their own specific domain. Therefore, according to Christie (1995), in functional tradition, Bernstein's (1990) pedagogic discourse is rephrased as pedagogic register. The specific discourse on which pedagogic texts are situated will be "pedagogic activity" in which "the purposes of teaching and learning" have been relocated (Christie, 1995, p. 223).

In order to avoid confusion, pedagogic discourse that is used in the succeeding sections of this study will follow a functional description of discourse, such as the texts used for curriculum purposes in early childhood as provided by Christie (1995). One pedagogic discourse is a socially constructed activity realised through language and other semiotic resources in the curriculum. The scope of pedagogic discourse covers from the smallest unit of analysis such as one table taxonomy, one question, or one textbook in which the pedagogic purposes are realised, to the comparatively larger pedagogic documents such as one syllabus, one curriculum guideline or one textbook. The smallest unit is termed a pedagogic item while the comparatively larger pedagogic documents are termed pedagogic discourse. The relationship between pedagogic item and pedagogic discourse is that one pedagogic item is "a segmental construal of reality" (Martin, 2012, p. 83). The overall social purposes of pedagogic discourse are maintained and preserved in the pedagogic item.

3.5 Curriculum ecology: identifying pedagogic discourses in one curriculum

A model of the curriculum ecology of mathematics is presented in Figure 3.5. This model captures the provisional interactions between different pedagogic discourses operating at the curriculum macro-genre level. Pedagogic discourses identified within one curriculum (in this case, mathematics) will be syllabus, curriculum guideline, one series of mathematical textbooks, auxiliary materials accompanying mathematical textbooks, classroom discourses, and examinations. Halliday's concerns about "every aspect of language and learning", areas in language education such as "teacher training, curriculum and syllabus construction, instructional language of the classroom and the socio-cultural aspects of language teaching" (Halliday, 2007, p. 239) are all involved in designing a language subject curriculum. Halliday's (2007) idea informs the emergence of curriculum ecology dealing with "a segmental construal of reality" (Martin, 2012, p. 83) which could be further segmented until the smallest item.

In this case, the smallest item is pedagogic item that demonstrates "a schematic outline of the textual strategy" (Martin, 2012, p. 95).

The delicate segmentation of pedagogic discourse in line with the rank scale offers a holistic picture in order to see how the information within one pedagogic discourse unfolds geographically and/or temporally. By referring to geographically, pedagogic discourses in written form are considered. For example, syllabus, curriculum guideline, one series of mathematical textbooks, auxiliary materials accompanying mathematical textbooks, and examinations are all unfolding as dependent on the physical affordances such as paperback booklets or digitalized electronic documents. By referring to temporally, pedagogic discourses that are in spoken form are considered. In this case, classroom discourse is the most typical type of pedagogic discourse predominant in spoken format (e.g. teacher's talk, teacher and student interactions, student talk). This pedagogic discourse is sequenced in temporal order: classroom discourse for mathematics at secondary level is serialized into a large number of individual lessons across six years of secondary school education. Apart from the predominant spoken forms, written discourses such as textbooks, lessons notes, and the like also appear in classroom settings, contributing to the complexity of classroom discourses.

This curriculum ecology captures the network relationship between different pedagogic discourses in producing the outcome of education. This network is underlined by using Bernstein's (1990) recontextualisation with linguistic affordance from systemic functional semiosis: the use of systemic functional linguistics in understanding semiotic resources in addition to language.

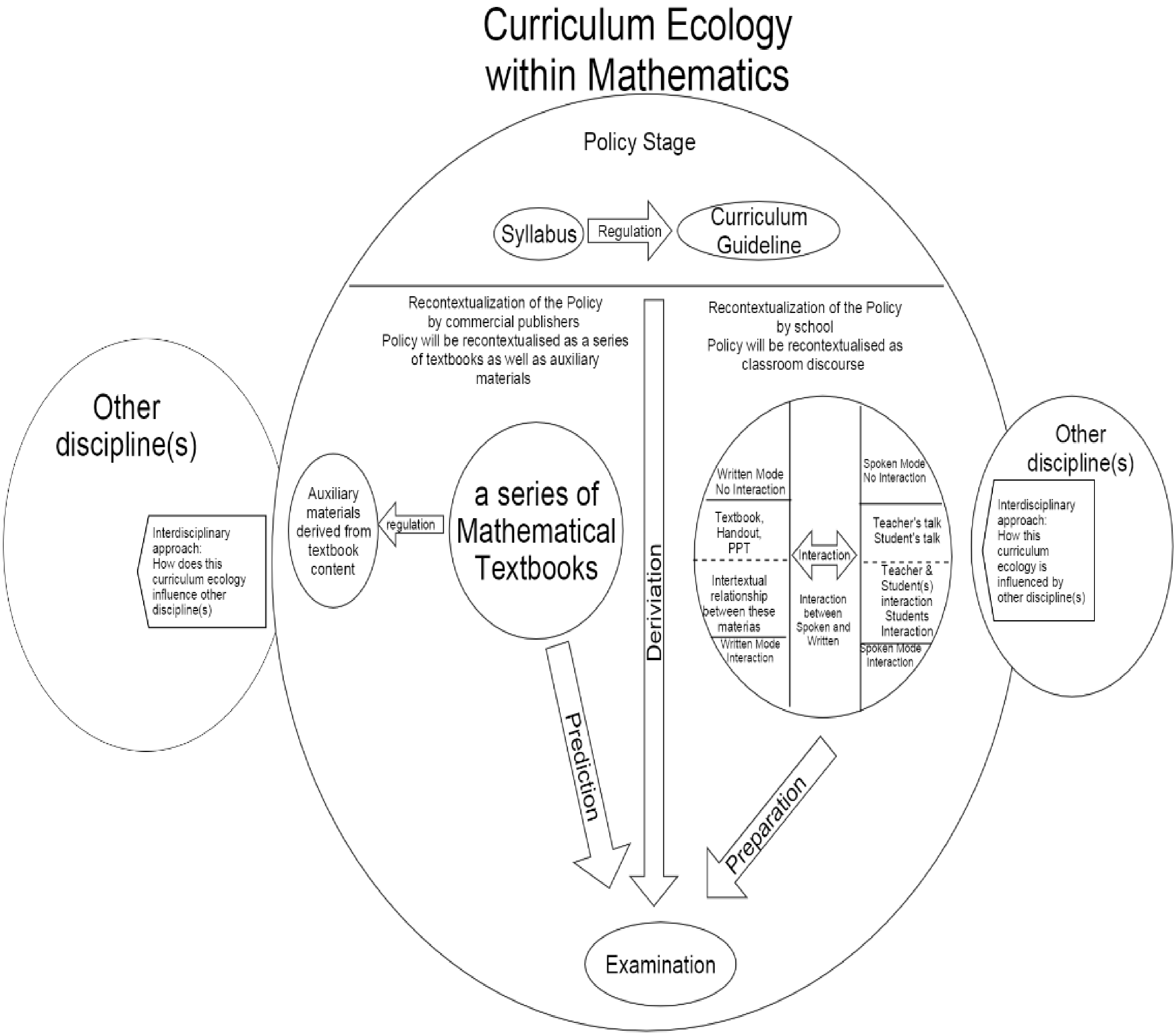


Figure 3.5: Curriculum ecology within mathematics

In Section 3.6, I suggest that recontextualisation is the underlying principle driving the mobilization of curriculum ecology.

3.6 Recontextualisation as the underlying principle driving the mobilization of curriculum ecology

In order to capture a preliminary understanding of the relationship between different pedagogic discourses, a model of curriculum ecology is proposed in Section 3.7. This curriculum ecology depicts the relationship between existing pedagogic discourses regarding how the curriculum progresses in the following three sectors: policy-making, policy implementation, and interdisciplinary influences. It is proposed that recontextualisation is the underlying principle helping to bridge different pedagogic discourses. With the help of recontextualisation, the curriculum ecology could be interpreted. As defined by Bernstein (1990), it is the "recontextualising principle" that "transforms the actual into the virtual or imaginary" and it is "a signifier for something other than itself" (p. 183). Termed "recontextualisation" (Bernstein, 1990, p. 184), the transformation between different pedagogic discourses will be realised when one discourse "appropriates, relocates, refocuses and relates to other discourse" (Bernstein, 1990, p. 159). The following quotation from Bernstein (1990) provides a sociological interpretation about recontextualisation between pedagogic discourses.

> [Recontextualisation between pedagogic discourses] is a principle which removes (delocates) a discourse from its substantive practice and context, and relocates that discourse according to its own principle of selective re-ordering and focusing. In this process of the delocation and the relocation of the original discourse the social basis of its practice, including its power relations, is removed. In the process of the de- and relocation, the original discourse is subject to a transformation that transforms it from an actual practice to a virtual or imaginary practice. (p. 159)

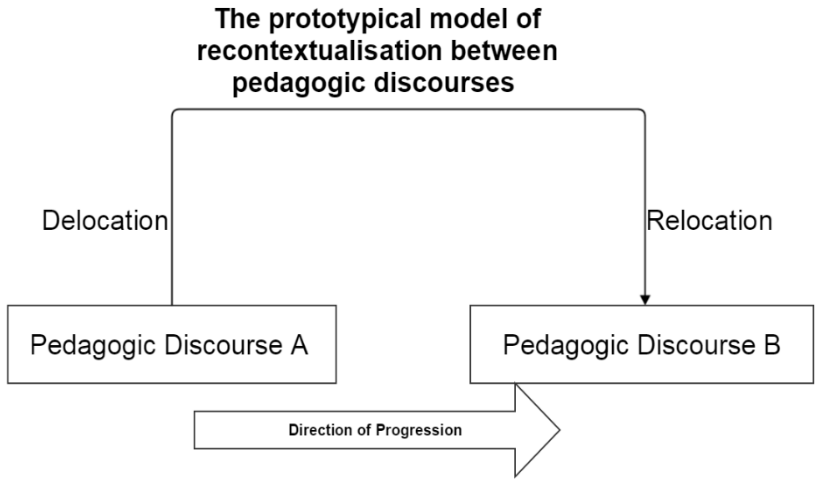


Figure 3.6: Prototypical model of recontextualisation between pedagogic discourses

Bernstein's (1990, p. 159) explanation of recontextualisation was presented in Figure 3.6. In that model, a proto-typical example of how recontextualisation occurs during the transmission between two pedagogic discourses is visualized. The point of departure starts from Pedagogic Discourse A while Pedagogic Discourse B is the recontextualised outcome of Pedagogic Discourse A. "Internal characteristics" (Bernstein, 1990, p. 159) of Pedagogic Discourse A is delocated from their semantic representations. During the process of relocation, Pedagogic Discourse B is therefore composed, within which these internal characteristics have been relocated. The transformation between Pedagogic Discourse A and Pedagogic Discourse B signals that these two activities are essentially sharing the internal characteristics that allow these two activities to be connected.

Bernstein's (1990, p. 159) notion of recontextualisation has been wildly adopted in the field of education. For example, Christie and Derewianka (2008, p. 151) found that recontextualisation is a "feature of all school teaching and learning". It suggests how one pedagogic discourse was converted into others. Incidents indicating "pedagogical recontextualization of meaning" are for pedagogical purposes and this "recontextualization of discourses for pedagogical purposes does reflect the values and ways of thinking of the disciplinary communities" (Schleppegrell, 2004, p. 114). For example, according to Rose (1997, p. 71), the disciplinary communities consisting of various education stakeholders such as "pedagogy theorists, teachers, educators and curriculum and textbook writers"

(1997, p. 71) are “recontextualisers” (Bernstein, 1990, p. 188) participating in different types of recontextualisation of the curriculum ecology. “The values and ways of thinking” (Schleppegrell, 2004, p. 114) conceived by different “recontextualisers” (Bernstein, 1990, p. 188) have been embedded within pedagogic discourses, suggesting that recontextualisation is purpose-oriented.

The same recontextualiser might also produce different pedagogic discourses for different purposes. For example, in order to ensure disciplinary knowledge learnable by every child, the same recontextualiser such as teachers and textbook writers will produce different pedagogic discourses (Christie & Derewianka, 2007). For example, although both lengthy descriptions and a summary have been adopted to account for the same technical concept, their purposes are different with the former proceduralizing a series of correlated concepts through procedural genre and the latter summarizing these correlated concepts into a terminology. Through the recontextualisation, students will develop the skills to “manipulate often unfamiliar technical knowledge” (Christie & Derewianka, 2007, p. 151).

Within mathematics, where this study is situated, the relationship between different pedagogic discourses in the curriculum ecology is interpreted from the recontextualization perspective while one pedagogic discourse is the recontextualized outcome of the other. For example, at the policy-making stage, syllabus (EDB, 1999) and curriculum guidelines (HKEAA, 2007) are pedagogic discourses which established the overall structure for the curriculum of Mathematics in Hong Kong. The aims, objectives and structure of mathematics, together with teaching and learning objectives in each module and unit have been underlined in the Syllabus (EDB, 1999, p. 1). These underlying features correspond to the notion of “internal characteristics” proposed by Bernstein (1990, p. 159). Internal characteristics of the Syllabus (EDB, 1999) have been delocated in order to compose other pedagogic discourses in the same curriculum ecology. The first pedagogic discourse that accomplished the recontextualisation from the Syllabus (EDB, 1999) is the curriculum guideline (HKEAA, 2007). Preserving the internal characteristics embedded within the Syllabus (EDB, 1999), curriculum guideline (HKEAA, 2007) is a recontextualised version of the Syllabus (EDB, 1999) at the policy-making stage immediately after the education policy in Hong Kong was shifted from a 3-4-3 system to a 3-3-4 system. Influenced by the education reformation, the appearance of the Mathematical Curriculum Guideline in 2007 drove the education system in Hong Kong to be accommodated with the new 3-3-4 system. Based on that system, the period of secondary school education has been reduced from seven years to six years while the tertiary school education has been extended from three years to four years. In the secondary school mathematics curriculum, this transition has been reflected in the reduction in the scope of a range of mathematical concepts such as *Calculus* from the existing syllabus published in 1999 (EDB, 1999). As a result, this reduction resulted in a series of changes in the composition of other pedagogic discourse. For example, the popular textbooks that are now currently used in Hong Kong Secondary schools were all produced after the year 2007, marking the publishers' effort to adjust to the new policy. The curriculum guideline (HKEAA, 2007) is a reflection of this new system. It quotes what has been maintained in the syllabus (EDB, 1999), restates the significance of the mathematics curriculum in secondary school education, reiterates the controlling effects from the syllabus (EDB, 1999) in designing other pedagogic discourses and explicates the teaching strategies of several key mathematical concepts. In essence, the curriculum guideline is an extended version of the syllabus (EDB, 1999) within which the newly deployed education system in Hong Kong could be reflected at the policy-making stage.

During the policy implementation period, pedagogic discourses at the policy making-stage have been recontextualized by a variety of education stakeholders, leading to different types of pedagogic discourses when the ecology progresses. These education stakeholders could be categorized into three major classes: textbook publishers, schools and the education authority. Following Bernstein's (1990) classification, these education stakeholders are recontextualisers in the curriculum of Mathematics in Hong Kong who delocate and/or relocate the internal characteristics of mathematics when the curriculum ecology progresses.

To be more explicit, the implementation of mathematical education policy initially develops into three major sectors all of which undergo the process of recontextualisation: the recontextualisation by textbook publishers, the recontextualisation by each school and the recontextualisation by the education authority. Textbooks are primary resources in which the policy has been recontextualised by textbook publishers. The recontextualisation by textbook publishers could be serialized as: 1) policy had been comprehended by textbook publishers; 2) their comprehension was materialized as textbooks because textbook publishers produced textbooks based on their understanding of the policy; 3) auxiliary materials such as extracurricular exercises created by the same publisher were then produced based on textbooks. The other recontextualiser who also contributes to the recontextualisation of the policy is the schools. For example, the recontextualisation of the policy by a typical local Hong Kong school could be serialized as: 1) policy should be comprehended by a disciplinary panel consisting of subject teachers; 2) the comprehension will be consolidated as the teaching plan; 3) guided by the teaching plan, each subject teacher will design his or her own teaching in the classroom. Pedagogic practices involved in this type of recontextualisation include, but are not limited to, the lesson plan, teacher's talk, students' talk, teacher-student interaction, Power-Point slides, in-class quiz and handouts, amongst others. With the development of new media and new technology, it is predicted that more types of pedagogic practices will emerge in the classroom as new forms of recontextualisation of the policy. The third type of recontextualisation performed by the education authority leads to one crucial part on the curriculum ecology; the examination. Being the same designing body of both Syllabus (EDB, 1999) and Curriculum guides (HKEAA, 2007), the Hong Kong Examinations and Assessment Authority is the education authority that is responsible for preparing the large-scale yearly mathematics examination held in Hong Kong each year since 2012. Quoting from examination papers (HKEAA, 2012 & 2013), "knowledge of the mathematical facts, concepts, skills and principles" presented in the Syllabus (EDB, 1999) and the Curriculum and Assessment Guide (HKEAA, 2007) will be recontextualised as mathematical questions to test candidates for the Hong Kong Diploma of Secondary Education examination. Hence, at this stage, the internal characteristics at the policy stage have been relocated as sets of mathematical questions.

The curriculum ecology in Figure 3.5 also reflects some new advances in the school curriculum such as the emergence of an interdisciplinary approach. This interdisciplinary approach addresses two complementary aspects: first, how the mathematical discipline is influenced by other disciplines, and second, how the mathematical discipline influences other disciplines. The involvement of an interdisciplinary approach into the curriculum ecology is suggestive of studies in the field of interdisciplinary research where individual discipline is intended to be associated with others rather than in isolation. In the field of mathematics, this interdisciplinary approach has been identified in the dialogue between mathematics and physics, between mathematics and chemistry and between mathematics and economics, among others, where mathematical knowledge and skills have been recurrently utilized to underline physical, chemical and economic equations, and theorems at the level of secondary schooling. This recurrent utilization of mathematical knowledge in other disciplines signals the existence of recontextualisation where parts of mathematical knowledge as the internal characteristics have been delocated from mathematics and then relocated into other disciplines. Mathematics has also been influenced by disciplinary knowledge of English and Chinese in cases where verbal language is the primary resource construct of mathematical deductions and argumentations (O'Halloran, 2005). If the mathematical discourse is written in English, the disciplinary knowledge of English will be delocated and relocated in constructing mathematical discourse, especially the grammar of the verbal texts in mathematical discourse.

3.7 Summary of chapter

Following the discussion of the mathematical knowledge structure and the relationship between knowledge and representation in Chapter Two, this chapter offered some theoretical considerations that will be useful for further analysis. It began with a discussion of the viewing language as social semiotic resources, proposing three parameters, that dealt with the semiotic explorations of language. These three parameters are realisation, individuation and instantiation each with its own focus on language. This chapter then argues the criteria of pedagogic discourse, reconciling the sociological perspective with the social-semiotic account. Pedagogic discourse is concerned as a socially constructed activity realised through language and other semiotic resources in the curriculum. A framework called curriculum ecology is proposed by me where different types of pedagogic discourses are connected. Their connection is mobilized through recontextualisation. The underlying relationship between different pedagogic discourses on the curriculum ecology could be argued from the perspective of instantiation once we treat each pedagogic discourse as a type of instantiation in which the mathematical knowledge is instantiated. Indeed, Halliday and Martin (1993) discussed the relationship between different instantiations in one text because "the way in which instantiations at one point in a text were conditioned by earlier instantiations" (Halliday and Martin, 1993, p. 51), addressing that the different instantiations within a text are interrelated to some extent. This notion that one instantiation is conditioned by earlier instantiations (Halliday and Martin, 1993) could be extended to cover the relationship between different pedagogic discourses that one instantiation at one point on the curriculum ecology is conditioned by earlier instantiations. The scope of text considered by Halliday and Martin (1993) has been therefore expanded to account for the curriculum.

Chapter Four
Research Paradigm, Research Gap, Research Focus, Research Questions and Research Methodology

4.1 Introduction

This chapter starts in Section 4.2 with the justification for the research paradigm adopted by this study. The Research questions and research gaps are outlined in Section 4.3. Section 4.4 introduces the various documents collected for this study. Section 4.5 specifies the research data identifying the representations of Pythagoras' Theorem. Section 4.6 proposes that recontextualisation is the trajectory of analysis and in Section 4.7, recontextualisation is reconciled with re-instantiation so that a linguistic approach is bridged with the sociological approach. In Section 4.8, an adapted model is provided of the commitment of ideational meaning in mathematical discourse. This model is an original framework proposed to analyse research data in the present study. In Section 4.9, examples of the research data are given, specifying the analytical procedure. Section 4.10 considers the ethical issues concerning the research data. Section 4.11 summarizes this chapter.

4.2 Research paradigm

Mathematical knowledge and the representation of mathematical knowledge, as two key notions to be discussed throughout this study, are elaborated from the perspective of social constructivism (Jørgensen & Phillips, 2002). Within this perspective, knowledge and its representations are differentiated. Knowledge exists in the conceptual domain while its representations exist in the reality domain. Figure 4.1 visualises the relationship between knowledge and representations.

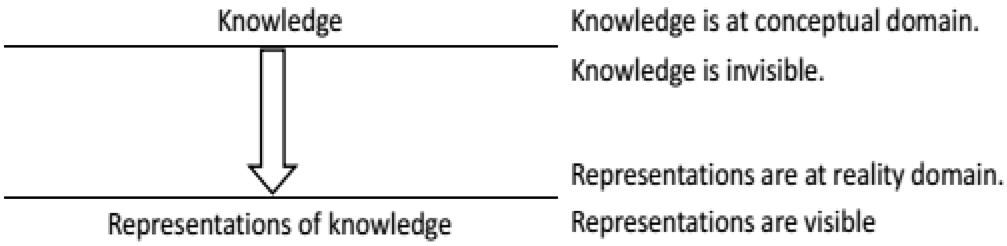


Figure 4.1: Relationship between knowledge and representations

In Figure 4.1, knowledge is presented in the conceptual domain, indicating that knowledge is invisible in nature. Representations of knowledge are presented in the reality domain, indicating that representations are visible. The arrow in Figure 4.1 stands for the process of representation. This process indicates that knowledge in the conceptual domain could be represented in the reality domain. Within the realm of mathematics, the representations of mathematical knowledge are the mathematical discourses constructed from mathematical semiotic resources, namely: verbal language, mathematical symbolism and visual images. These mathematical semiotic resources are investigated with the assistance of a Systemic Functional Multimodal Discourse Analysis (SFMDA) approach. In this approach, the major focus is to incorporate the semiotic resources into the discussion, rather than just verbal language.

4.2.1 Social constructivism of knowledge: knowledge and the representations of knowledge

As considered by Halliday and Matthiessen (1999), "all knowledge is constituted in a semiotic system" (p. 3). As discussed in Chapter Two, within the realm of mathematics, the representation of mathematical knowledge is achieved through mathematical discourses. Mathematical discourses are in turn organised by mathematical semiotic resources such as verbal language, mathematical symbolism and visual images. Mathematical knowledge and its representations are treated as separate items (Burr, 1995). Following a social constructivist perspective of knowledge building, the embodiment of the disciplinary knowledge of mathematics is represented in mathematical discourse. The representation of the knowledge is to represent it as the "semiotic system"

(Halliday & Matthiessen, 1999, p. 3) in the form of "products of our ways of categorising the world" (Jørgensen & Phillips, 2002, p. 5). As referred by Jørgensen and Phillips, these "products of our ways of categorising the world" (2002, p. 5) are the representations of knowledge organised by semiotic resources, echoing with the domain of reality in Figure 4.1.

This social constructivism perspective is the position held in this study. Following this perspective, as was mentioned at the beginning of this section, mathematical knowledge and the representations of mathematical knowledge have been separated from each other.

4.2.2 Systemic-functional multimodal discourse analysis approach (SFMDA approach)

Originating from Systemic Functional Linguistics (SFL) (Halliday & Matthiessen, 2004), the SFMDA approach is a theoretical consideration proposed to describe phenomena where semiotic resources involve more than just verbal language. In mathematics, as considered by O'Halloran (2005), semiotic resources such as mathematical symbolism and visual image work in line with verbal language. With respect to knowledge construction, Halliday and Matthiessen (1999) argue that the representation of knowledge is "constructed from language in the first place" (p. 3). With the development of multimodal and multi-semiotic studies, "language" in Halliday and Matthiessen's (1999, p. 3) notion could be extended to include multi-semiotic resources. From the standpoint of mathematics, as considered in this study, the representation of mathematical knowledge is achieved multisemiotically as well. Mathematical symbolism and visual image are also crucial resources in representing mathematical knowledge.

4.3 Research questions, research objectives and research gaps

The research question for this study is "how is mathematical knowledge transmitted and represented in different pedagogic discourses?" This question can be further expanded into three sub-questions:

1. How is mathematical knowledge represented in different pedagogic discourses?

2. How are the representations of mathematical knowledge connected to each other?

3. What is the relationship between different representations?

The aim of this study is to underline how mathematical knowledge is constructed in mathematical semiotic resources in various educational contexts. The contribution of this study is to identify how knowledge is represented in different semiotic resources and how these representations are connected to each other.

From a practical standpoint, a limited amount of research has focused on the relationship between knowledge and representation within a series of correlated educational documents. The present study intends to fill this gap supported by both theoretical and empirical evidence. The theoretical issues that this study deals with are the recontextualisation sites for knowledge development and dissemination. Driven by the findings from the study, I intend to create a research model that can be applied in other similar research areas where knowledge building and representation are considered in education fields. The educational purpose of this study is to help educational policy makers and textbook designers to find an applicable way to design educational documents with a more informed understanding of the relationship between knowledge and representation.

In order to accomplish the goals of this study, a qualitative data analysis approach will be implemented. The success of qualitative data analysis depends on whether the selected data is representative; therefore, studies conducted on similar data could benefit from my work. Sections 4.4 and 4.5 will discuss the procedure for screening and selecting appropriate research data in the present study. Section 4.4 first elaborates the importance of pedagogic discourses that are not used in classrooms in accomplishing academic purposes with reference to the complete curriculum ecology elaborated in Chapter Three. Qualified non-classroom pedagogic discourses, namely the Syllabus (EDB, 1999), Curriculum Guidelines (HKEAA, 2007), Examination Paper (HKEAA, 2012) and the mathematical textbook (Wong and Wong, 2009) will be justified based on their roles in the mathematical education system in Hong Kong. The selection of Wong and Wong (2009) as the mathematical textbook in this present study will be justified. This version of mathematical textbook is one of those recommended, and its quality positively assessed, by the EDB (2014).

Pedagogic discourses outlined in Section 4.4 will offer a platform from which research data related to Pythagoras' Theorem could be taken. In Section 4.5, the significance of focusing on Pythagoras' Theorem will be justified. With reference to the specific instances regarding this mathematical concept, places where Pythagoras' Theorem have been instantiated in the Syllabus (EDB, 1999), Curriculum Guidelines (HKEAA, 2007) and Examination Paper (HKEAA, 2012), will be provided in Appendices One to Three. As for the research data to be taken from Wong and Wong (2009), the selected instance of Pythagoras' Theorem is typical. Typicality could be understood from three points of view: first, the multi-semiotic resources used in this instance; second, a comparison with other mathematical textbooks that also instance Pythagoras' Theorem; third, a recognition that this instance is the point of departure from which recontextualisation will take place.

4.4 Selected pedagogic discourse in this study

The curriculum ecology for mathematics outlined in Figure 3.5 provides a provisional model for the relationship between different pedagogic discourses in the curriculum. Each pedagogic discourse works not in isolation but in conjunction with others. Two tiered structures, one for the rank scale of the pedagogic text, and the other for the rank scale of the genre structure, are elaborated and aligned to suggest that each pedagogic discourse is constructed based on a system of embedding. A list of pedagogic discourses has been provided to indicate the educational documents contained within one curriculum ecology. Due to limitations of time and space, this study concentrates only on part of the curriculum ecology of mathematics. This part is composed of the mathematical syllabus (EDB, 1999), curriculum guideline (HKEAA, 2007), the mathematical textbook and the HKDSE mathematical examination paper.

Figure 4.2 is an adaptation of the curriculum ecology outlined in Figure 3.5, focusing on the selected pedagogic discourses that this study will further investigate.

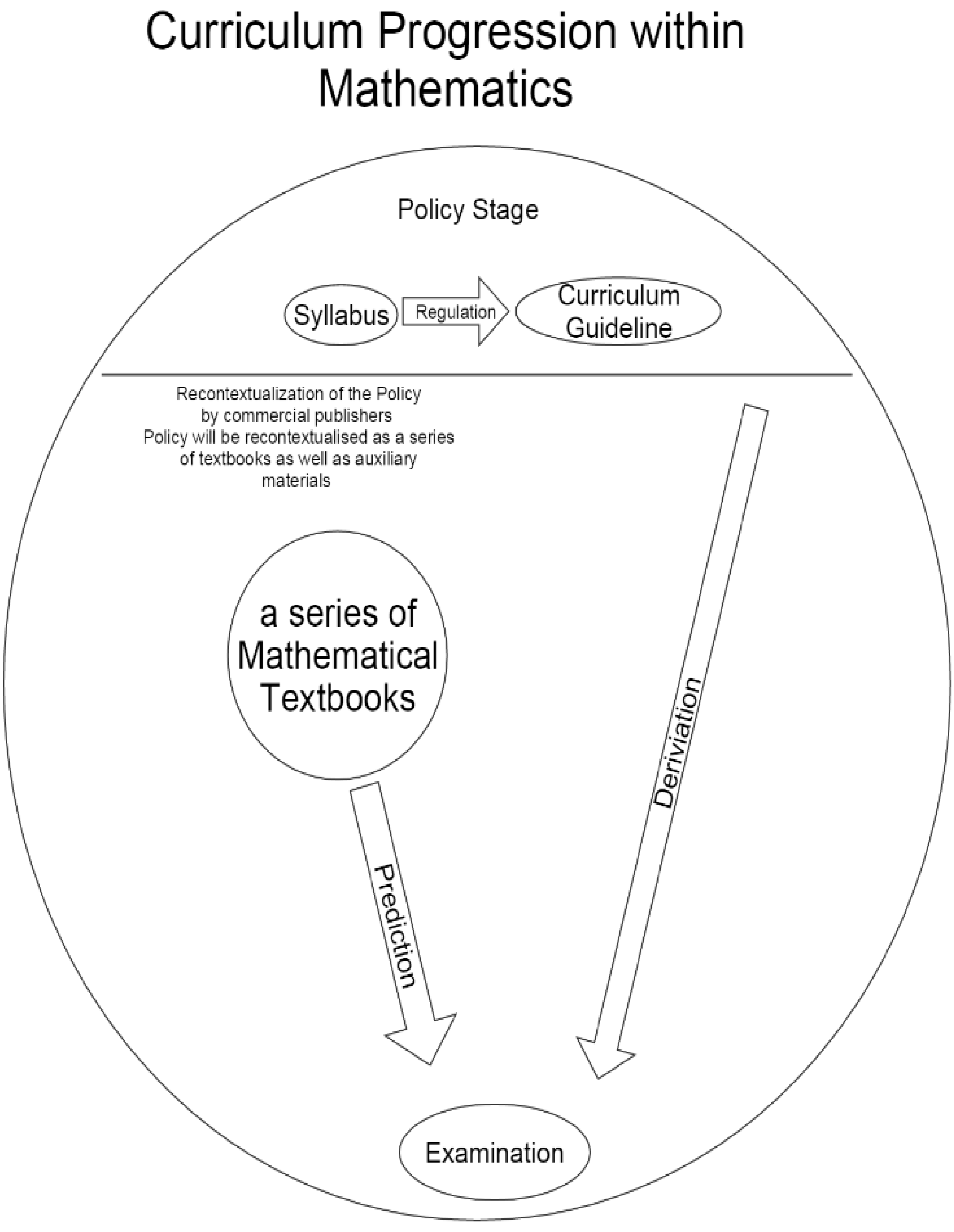

Curriculum Progression within Mathematics
Policy Stage
Syllabus
Regulation
Curriculum Guideline
Recontextualization of the Policy by commercial publishers Policy will be recontextualised as a series of textbooks as well as auxiliary materials
a series of Mathematical Textbooks
Deriviation
Prediction
Examination

Figure 4.2: An adaptation of the curriculum ecology in Figure 3.5: A pictorial representation of the partial curriculum ecology

Figure 4.2 is an adaption of Figure 3.5. Selected pedagogic discourses belong to the category of non-classroom pedagogic discourses. According to Bernstein (1990), “curricula cannot be acquired wholly by time spent at school” (p. 66). From a learner’s perspective, efforts should also be devoted to understanding how the curricula are acquired beyond classroom settings. However, rather than dismiss the crucial role played by classroom discourses in the dissemination and construction of knowledge in the education system, this study suggests that pedagogic discourses engaged in outside the school and classroom will be also important for mathematical knowledge building. These pedagogic discourses, which “go beyond practice” in the classroom setting (Billett & Choy, 2012, p. 158) have been positioned within the adapted curriculum ecology portrayed in Figure 4.2. Bernstein stated that these pedagogic discourses are “the crucial pedagogic medium and social relation” in educational contexts, as these pedagogic discourses help the student to “acquire the written code” (Bernstein, 1990, p. 46) emerging from the curricula. In this study, their significance goes beyond the facilitation of a written code. These pedagogic discourses play important roles in the dissemination and construction of knowledge. These pedagogic discourses include the Syllabus (EDB, 1999), Curriculum Guideline (HKEAA, 2007), the mathematical textbook and the examination paper (HKEAA, 2012). Research data regarding Pythagoras’ Theorem will be selected from these pedagogic discourses. Before going into the details of screening and selecting of research data regarding Pythagoras’ Theorem, the nature of these pedagogic discourses will be elaborated.

Regarding their source, the pedagogic discourses outlined in Figure 4.2 could be categorized into two streams: authority and commodity. Pedagogic discourses from the authority stream refer to the syllabus, curriculum guideline and examination paper as composed by education policy makers: That is, the syllabus is composed by the EDB, while both the curriculum guideline and examination paper are composed by the HKEAA. The relationship between these two policy makers is that the HKEAA is an affiliated section of the EDB. To justify the reasons for selecting pedagogic discourses from the authority stream, each of them plays a crucial role in the mathematics curriculum, reflecting the education system and ideology proposed by the Hong Kong government in implementing the secondary school education system. Specifically, mathematics was selected as one of the compulsory subjects for students. In accord with this aim, the syllabus attempts to scope and define the knowledge domain in which the conceptual network of the required mathematical concepts is connected. The syllabus also outlines the network structure between different mathematical concepts required at the HKDSE level. The Curriculum Guideline (HKEAA, 2007) adjusted the Syllabus (EDB, 1999) to accommodate the newly implemented education system. On the one hand, the Curriculum Guideline (HKEAA, 2007) preserves the network structure between different mathematical concepts outlined in the Syllabus (EDB, 1999); on the other hand, the Curriculum Guideline (HKEAA, 2007) enriched the Syllabus (EDB, 1999) with practical teaching and learning examples for key mathematical concepts. Promoted by the EDB, the mathematical examination at HKDSE level is a large-scale examination conducted in Hong Kong. As for the importance of the examination, the quotation from HKEAA (2007) is worth mentioning here.

> First, it (the examination) provides feedback to students on their performance and to teachers and schools on the quality of the teaching provided. Second, it communicates to parents, tertiary institutions, employers and the public at large what it is that students know and are able to do, in terms of how their performance matches the standards. Third, it facilitates selection decisions that are fair and defensible. (p. 129)

Based on the quotation above, it must be noted that the examination paper (HKEAA, 2012) is also a crucial pedagogic discourse in the education system. The Examination paper (HKEAA, 2012) is designed for the purposes of providing feedback on both teaching and learning and enabling future employers to assess students' performance after receiving HKDSE mathematical education.

The quality of the pedagogic discourses in the authority stream is guaranteed. Serial steps in the order of drafting, examining, assessing and re-examining formulate the essential procedure before these pedagogic discourses become published. The Editorial board for these pedagogic discourses is composed of experts in Mathematics such as academic staff at tertiary level, current secondary school mathematics teachers, principals and government officials. Each of the authority documents also invites comments and criticisms from the public for the further amendments when necessary. These prudent processes in composing authority documents ensure that these documents are representative of what is believed necessary for the advocacy of secondary school mathematics education in Hong Kong.

In terms of the commodity stream, mathematics textbooks are the pedagogic discourses that are crucial for both teaching and learning since "textbooks are the main resources that Mathematics teachers use in deciding what and how to teach" (HKEAA, 2007, p. 134). In terms of the range of commercial textbooks available in Hong Kong, the EDB provided the "List of Recommended Secondary School Textbooks in English" (EDB, 2014). On that list, six commercial textbook publishers[3] were recommended by the EDB as the qualified and shortlisted providers of mathematics textbooks suitable for HKDSE examination. Table 4.1 summarizes the recommended publishers and the names of their mathematics textbooks.

[3] These six commercial publishers are: Manhattan Press, Hong Kong Educational Publishing Company, Educational Publishing House Ltd, Oxford University Press (China) Ltd, Chung Tai Educational Press, and Pearson Hong Kong.

Table 4.1: Recommended publishers and mathematics textbooks

#	Publisher	Textbook Name	Author(s)	Year of Publication
1	Manhattan Press	Mathematics for Tomorrow	Chow Wai Man	2005
2	Manhattan Press	Discovering Mathematics	Chow Wai Keung	2009
3	Hong Kong Educational Publishing Company	New Progress in Junior Mathematics	H. M. Chan, W. H. Chan, Angus Cheng, K. T. Hung, C. K. Kwun, W. S. Lo, H. Y. Pang	2008
4	Educational Publishing House Ltd	Mathematics in Focus	Ho Mei Fun, Hung Chun Wah, Liu Wing Ki	2013
5	Oxford University Press (China) Ltd	Exploring Mathematics (Oxford / Canotta Maths Series)	Frederick Leung Koon Shing, Chu Wai Man, Fok On Ki, Luk Mee Lin	2005
6	Oxford University Press (China) Ltd	Oxford / Canotta Maths Series - New Century Mathematics	T.W. Wong, M.S. Wong	2009
7	Chung Tai Educational Press	New Trend Mathematics	Chan Mung Hung, Leung Shui Wah, Mui Wai Kwong, Kwok Pui Man	2008
8	Chung Tai Educational Press	Effective Learning Mathematics	Mui Wai Kwong, Chan Mung Hung, Tang Ming Chee, Lo Yin Kue, Milton T.Y. Lo, Tam Chi Fai	2015
9	Pearson Hong Kong	Mathematics in Action	P F Man, C M Yeung, K H Yeung, Y F Kwok, H Y Cheung	2009

In Table 4.1, nine textbook series published by six commercial publishers are recommended by the EDB to be used for the preparation of students for the HKDSE mathematics examination. Drawing from the curriculum ecology outlined in Figure 4.2, these mathematics textbooks listed in Table 4.1 are the recontextualised versions of both the Syllabus (EDB, 1999) and the Curriculum Guideline (HKEAA, 2007). Due to the limited time and space, only one series of mathematical textbooks is selected to be analysed, *Oxford/Canotta Maths Series – New Century Mathematics* published by Oxford University Press (China) Ltd. The reasons for this selection are twofold. First, the series of New Century Mathematics (S1–S6) is listed as one of the approved series of textbooks recommended by EDB (EDB, Textbook List, 2014) because this series of mathematics textbooks is closely associated with the learning targets required by the syllabus (EDB, 1999). Second, the publisher: Oxford University Press is aware that their series of New Century Mathematics (S1–S6) is constructed based on the EDB's (1999) requirements for publishers by indicating that their content matches what is required at HKDSE level. In terms of the quality assessment of this series of textbooks (Wong and Wong, 2009), EDB's evaluation is worth quoting here (EDB, Textbook List, 2014, p. 1):

- Meeting the aims and objectives of the curriculum guide
- Content self-contained and effective in meeting the curriculum requirements
- Generally appropriate learning activities facilitating achievement of the learning targets on the whole
- Precise and accurate use of language

Based on the above evaluation provided by the Textbook Committee at the EDB, the series of New Century Mathematics (S1–S6) (Wong & Wong, 2009) received positive feedback. This will ensure the appropriateness of data from the commodity stream for analysis, reflecting the closeness of content to the learning objectives set out in syllabus, as well as its precision and accuracy in the use of English to design the materials.

Following the justification of pedagogic discourses concerned in this section, Table 4.2 provides a list of pedagogic discourses based on the curriculum ecology.

Table 4.2: Types of pedagogic discourses concerned in this section

Staging Curriculum Ecology	Pedagogic texts outside the classroom settings	Source of data
Policy Stage	Mathematics Syllabus	EDB (1999)
	Mathematics Curriculum guideline	HKEAA (2007)
Implementation Stage	Mathematics textbooks	Wong and Wong (2009)
Assessment Stage	Mathematics Examination Paper at HKDSE level	HKEAA (2012)

Table 4.2 summarises the types of pedagogic discourses that play crucial roles as activities outside the classroom setting. Since there are altogether 136 key mathematical concepts required for study at the HKDSE level, it will be impossible for me to analyse all of them. The next section will narrow the research focus to one mathematical concept, namely Pythagoras' Theorem. The findings that emerge from the analysis of Pythagoras' Theorem can be generalised to apply to other mathematical concepts.

4.5 Specifying Pythagoras' Theorem

Among all the mathematical concepts that are included in the syllabus at HKDSE level, Pythagoras' Theorem has been selected as the focus of the present study. The reasons for this particular selection are threefold: first, as a mathematical concept, Pythagoras' Theorem forms one of the compulsory learning objectives for all students aiming at the HKDSE mathematics examination. This compulsory component accounts for the recurrent occurrence of Pythagoras' Theorem in different pedagogic discourses in the curriculum ecology. Second, Pythagoras' Theorem is one of the ten greatest equations of all time in the field of both mathematics and physics (Crease, 2004). The influence of Pythagoras' Theorem enables the discussion in this study to remain influential and essential to the field in future pedagogic usage. Last but not the least, Pythagoras' Theorem is regarded as the most controversial mathematical theorem in the history science and mathematics in terms of whether this Theorem should be named after Pythagoras or "Gou Gu Theorem". One of the findings to be emerging from this

Phd Project is intended to answer this question. To move one step forward, the answer will be an exploratory one and could be applied in solving the "Needham's Grand Question"[4].

For the sake of specifying Pythagoras' Theorem, pedagogic discourses outlined in the previous Section 4.4 will undergo a screening process. Qualified instances selected from each pedagogic discourse should be representative and will comprise the research data to be investigated in this study. Following the embedded nature of pedagogic discourse outlined in Chapter Three, selected instances where Pythagoras' Theorem is mentioned should be typical instances because the success of qualitative analysis depends on "the quality of exemplars chosen for the analysis" (Martin & Rose, 2014, p. 313). A qualitative analysis conducted on these pedagogic discourses will unveil how Pythagoras' Theorem develops logo-genetically within the activities concerned in the partial curriculum ecology in Figure 4.2.

With respect to the research data selected from the authority stream, the specific locations of Pythagoras' Theorem are nominated below:

- Syllabus: EDB, 1999, p. 13; EDB, 1999, p. 23; EDB, 1999, p. ANNEXX III
- Curriculum Guidelines: HKEAA, 2007, p. 113
- Examination Paper: HKEAA, 2012, p. 6

With respect to the research data selected from the commodity stream, one instance exemplifies how Pythagoras' Theorem is typically introduced in the textbooks as a technical term, taken from Wong and Wong (2009, p. 103).

[4] In his classic book *The Grand Titration: Science and Society in East and West*, Needham (1969) proposed the following question: "Why, then, did modern science, as opposed to ancient and medieval science (with all that modern science implied in terms of political dominance), develop only in the Western world?" (p. II).

In a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of two adjacent sides.

i.e. In ΔABC, if $\angle C=90°$,

then $a^2+b^2=c^2$.

(Abbreviation: Pyth. theorem)

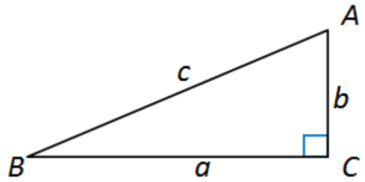


Figure 4.3: Selected example (Wong & Wong, 2009, p. 103)

Pythagoras' Theorem has been introduced similarly in each of the nine recommended textbooks, and Appendices 4 to 13 display the variants. Authentic semiotic resources are preserved, including the frame, the colour and the icons together with the three mathematical semiotic resources. An artificial line will be drawn to separate mathematical semiotic resources from others such as colours and icons. This separation is deliberate in order to give extra room for the discussion of semiotic resources other than mathematical semiotic resources. Therefore, Appendices 4 to 12 will be translated into those provided from Figures 4.3 to 4.11, with verbal language, symbolic equation and visual diagram preserved for the purposes of arguing how Pythagoras' Theorem has been instantiated in one technical term.

In a right-angled triangle, the sum of the squares of the two shorter sides *a* and *b* is equal to the square of the hypotenuse *c*. It is known as the **Pythagoras' Theorem.**

i.e. $a^2+b^2=c^2$

(Short form: Pyth. Theorem)

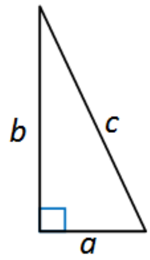


Figure 4.4: Instance of Pythagoras' Theorem selected from Chow (2005, p. 220)

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the square of the square of the other two sides

(Abbreviation: Pyth. Theorem)

That is:

in ΔABC, if $\angle C=90°$

then $a^2+b^2=c^2$ (Pyth. Theorem)

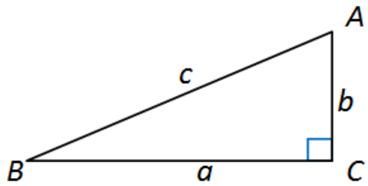


Figure 4.5: Instance of Pythagoras' Theorem selected from Chow (2009, pp. 11–13)

In a right-angled triangle, the sum of the squares of the two shorter sides is equal to the square of the hypotenuse.

That is: in ΔABC, if $\angle C=90°$

then $a^2+b^2=c^2$

(Reference: *Pyth. theorem*)

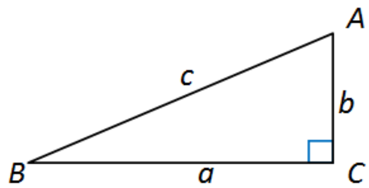
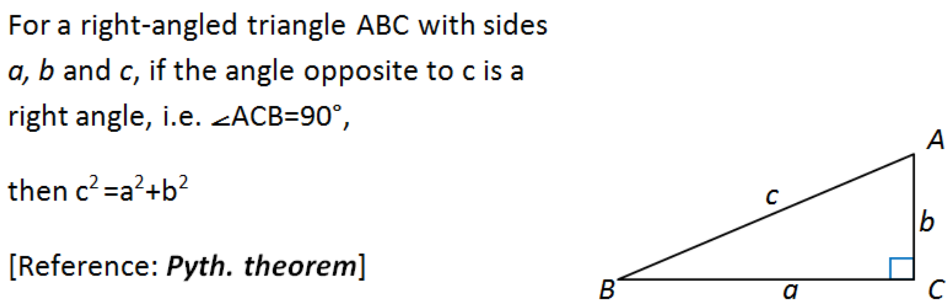


Figure 4.6: Instance of Pythagoras' Theorem selected from Chan et al. (2008, pp. 10–12)

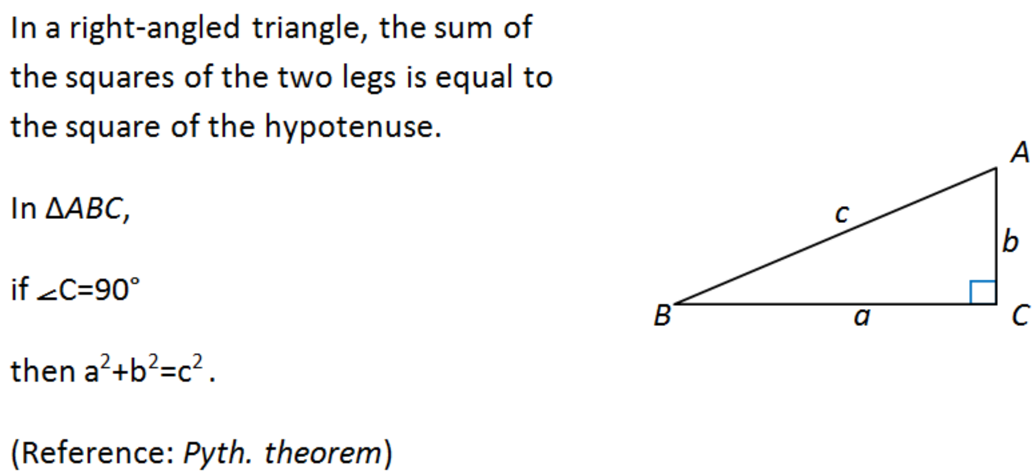


For a right-angled triangle ABC with sides *a, b* and *c*, if the angle opposite to c is a right angle, i.e. ∠ACB=90°,

then $c^2=a^2+b^2$

[Reference: ***Pyth. theorem***]

Figure 4.7: Instance of Pythagoras' Theorem selected from Ho et al. (2013, p. 98)



In a right-angled triangle, the sum of the squares of the two legs is equal to the square of the hypotenuse.

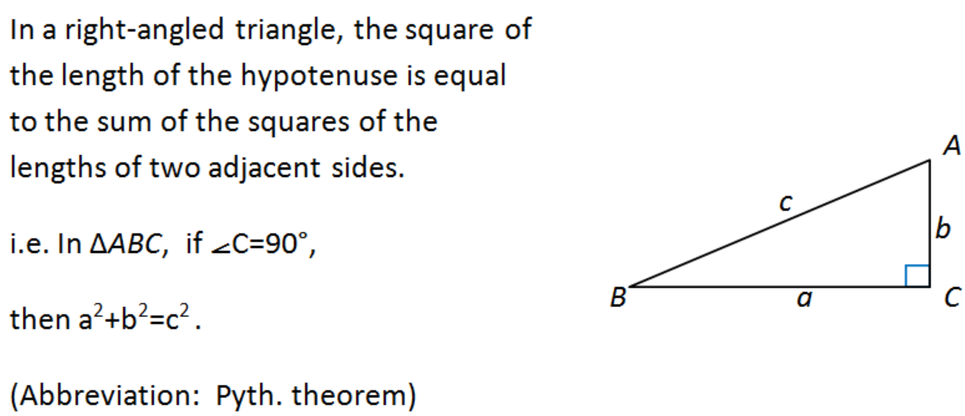
In Δ*ABC*,

if ∠C=90°

then $a^2+b^2=c^2$.

(Reference: *Pyth. theorem*)

Figure 4.8: Instance of Pythagoras' Theorem selected from Leung et al. (2005, p. 80)



In a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of two adjacent sides.

i.e. In Δ*ABC*, if ∠C=90°,

then $a^2+b^2=c^2$.

(Abbreviation: Pyth. theorem)

Figure 4.9: Instance of Pythagoras' Theorem selected from Chan et al. (2008, p. 69)

In a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of two adjacent sides.

I.e. In ΔABC, if $\angle C=90°$,

then $a^2+b^2=c^2$.

(Abbreviation. Pyth. Theorem)

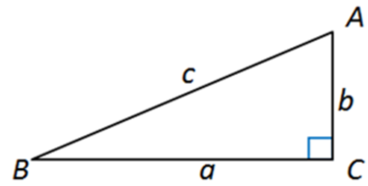


Figure 4.10: Instance of Pythagoras' Theorem selected from Mui et al. (2015, p. xi)

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two shorter sides.

i.e. In ΔABC,

if $\angle C=90°$,

then $a^2+b^2=c^2$.

(Abbreviation: Pyth. theorem)

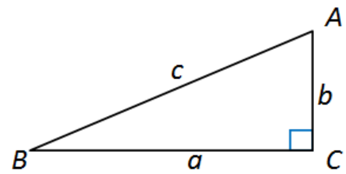


Figure 4.11: Instance of Pythagoras' Theorem selected from Man et al. (2009, p. 12)

The importance of using this instance as the selected data is also because this instance could be treated as the point of departure from which the knowledge of Pythagoras' Theorem could be "retheorized" (Halliday, 2004, p. 46) in other contexts, such as in "concrete categories" or in "'uncommonsense terms'" (Halliday, 2004, p. 46). This property of re-theorization could be bridged with recontextualisation, preparing recontextualisation as the trajectory into the analysis of this study.

The version selected from Wong and Wong (2009) will be analysed. From the perspective of the semiotic construction of this example, it is a semiotic complex where verbal language, mathematical symbolism and visual image co-exist to make meaning. From the perspective of the layout of this instance, information arrangement follows a pattern where lengthy description precedes a symbolic equation, a technical term summarises the information beforehand and a visual image is aligned and juxtaposed with other components. This pattern of information organisation will be discussed in more detail in Chapter Five. Another section in Chapter Five will be devoted to a discussion of how the meaning-making process in Figure 4.3 could be replicated to account for the meaning-making process in other similar instances (e.g. Figures 4.4 to 4.11).

4.6 Recontextualisation: a trajectory into the analysis

The purpose of this study is to understand how mathematical knowledge is represented in different pedagogic discourses in the curriculum ecology and to understand how these pedagogic discourses are mobilized with the help of recontextualisation. The nature of mathematical knowledge and the position held by this study towards the underlying structure of mathematical knowledge have been discussed in Chapter Two. To reiterate, in the present study mathematical concepts are treated as mathematical knowledge. Recontextualisation, as considered by Bernstein (1990), is a rule in educational contexts that "selectively appropriates, relocates, refocuses, and relates other discourses to constitute its own order" (p. 184). Following Bernstein's (1990) notion, Linell (1998) offers a long list of what can be recontextualised in the communication between different texts, including "linguistic expressions, concepts and propositions, facts, arguments and lines of argumentation, stories, assessments, values, and ideologies, knowledge and theoretical constructs, ways of seeing things and ways of acting toward them, ways of thinking, and ways of saying thing" (p. 145). Regarding the theoretical foundation of recontextualisation, Linell's (1998) list, although providing some specific examples where recontextualisation occurs, does not automatically generate a theorized framework because, as challenged by Kong (2008), "there is no framework that can tie in nicely with all of these concepts" (p. 436). However, Linell's (1998) list did offer a way of thinking about what can be recontextualised. For example, knowledge is a key component that could be recontextualised from one text to another. Linell also recasts Bernstein's (1990) description of recontextualisation, treating recontextualisation as "the dynamic transfer-and-transformation of something from one discourse/text-in-context (the context being in reality a matrix or field of contexts) to another" (Linell, 1998, p. 145). Linell's (1998) theorization of recontextualisation pushes forward a theoretical consideration guided by which recontextualisation could operate at a discourse level. Drawing on Linell's (1998) long list and Bernstein's (1990) notion of recontextualisation, in this study, mathematical knowledge is the entity that has been selectively recontextualised in different pedagogic texts. In order to understand "the 'what' and the 'how' of any transmission" (Bernstein, 1990, p. 56), issues regarding "what is relayed" and "how the contents are relayed" (Bernstein, 1990, p. 56) are considered. Knowledge is the pivot around which all the semiotic resources are constituted. Pedagogic texts, constituted through a range of semiotic resources, are relayed one after another, centrally because of their shared knowledge.

The recontextualisation between different pedagogic discourses entails what Bernstein termed "a relay" (Bernstein, 1990, p. 55) indicating the transmission between them. In the present study, curriculum ecology has been treated as an ideal education system where there is a relay of knowledge between them. According to Bernstein (1990), the relationship between different pedagogic discourses "is simply a relay for something other than itself" (p. 146). Using a sports metaphor, Pythagoras' Theorem is the baton in the relay race. With respect to the selected data (EDB, 1999, HKEAA, 2007, Wong and Wong, 2009 and HKEAA, 2012), there is relay of knowledge between different instances of Pythagoras' Theorem. These instances are correlated with others, since they realise the same mathematical knowledge. They are also different from each other since each of the instances is an individual representation of the same mathematical knowledge. The next section will bring re-instantiation into this study to describe recontextualisation from a linguistically informed approach.

4.7 Reconciling Recontextualisation with Re-instantiation

This section reiterates that recontextualisation is an applicable model in understanding the progression and representation of knowledge in a series of correlated pedagogic discourses. Reinstantiation (Hood, 2008) will be incorporated with recontextualisation to equip it with linguistic considerations.

This is the original framework that will be used for data analysis. Commitment (Painter et al., 2013), which is the method of underpinning re-instantiation is also provided. As articulated in Chapter Three and this chapter, Section 4.4, curriculum ecology is a network in which different pedagogic texts within a curriculum coordinate with each other. The orchestration is mobilised through recontextualisation, which assists the reformation between different pedagogic texts. In terms of the research data in this study, different pedagogic texts are instances where Pythagoras' Theorem has been represented.

The basic unit of analysis is a pedagogic item (see p. 68 of this Phd thesis). The core step in modelling the relationship between knowledge and representation is to identify how the knowledge of Pythagoras' Theorem is represented within one pedagogic item. Empowered by recontextualisation, the prior focus is to look at the delocation and relocation of Pythagoras' Theorem within one pedagogic item. Recontextualisation needs to be consolidated from a linguistic approach through which the relationship between knowledge and the representation of knowledge could be modelled. This linguistic approach is systemically and functionally informed – re-instantiation. This section reconciles recontextualisation with re-instantiation, providing recontextualisation with linguistic models to account for the representation of Pythagoras' Theorem in different pedagogic texts. This framework I designed in this present study is an innovative and original approach designed to understand knowledge construction in education.

4.7.1 Reinstantiation and serial reinstantiation

This study is theoretically underpinned as an SFL-informed research (Halliday & Matthiessen, 2004). As has been outlined in Chapter Three, instantiation is selected as the parameter for analysis among the three complementary parameters. The other two parameters are realisation and individuation. Relying on Halliday and Matthiessen's (2004) analogy between system and language, the collection of the pedagogic texts about Pythagoras' Theorem is not just the sum of all possible texts in the education system regarding Pythagoras' Theorem. This collection provides "an explanatory power" (Halliday and Matthiessen, 2004, p. 27) to answer how the system has been instantiated as different instances regarding Pythagoras' Theorem. There is an analogous relationship between a complete range of representations of Pythagoras' theorem and one specific instance of Pythagoras' Theorem. Generally, this relationship could be seen as the relationship between system of representations of a specific mathematical concept and one specific instance of the representations at the semiotic level because "these patterns of instantiation show up quantitatively as adjustments in the systemic probabilities of a language" (Halliday & Matthiessen, 2004, p. 27). In the present study, the knowledge of Pythagoras' Theorem could be viewed as "the meaning potentials as a whole" (Painter et al., 2013, p. 134) while different instances of Pythagoras' could be viewed as different particular instances. Drawing from the curriculum ecology outlined in Figure 3.5, the particular representations of Pythagoras' Theorem will be the pedagogic texts that are not used in a classroom setting. These pedagogic texts instantiate the meaning potential of Pythagoras' Theorem in the specific scope: how Pythagoras' Theorem has been instantiated in pedagogic texts other than classroom settings. Although more work could be dedicated to underline how Pythagoras' Theorem has been instantiated in classroom settings, such work would be beyond the scope of the present study.

The central question concerned with reinstantiation is "how does the meaning potential of one differ from the meaning potential of the other?" (Hood, 2008, p. 356). The change between different texts represents "the serial re-instantiation" (Hood, 2008, p. 352) from one text to another. In this study, this "serial re-instantiation" (Hood, p. 352) could be identified in the selected research data: syllabus, curriculum guideline, textbook and examination paper, each occupying one key place in the curriculum ecology and they together are internally mobilised through recontextualisation.

A logo-genetic unfolding was originally associated with "the development of a single text" (Hood, 2008, p. 351) through understanding how each text unfolds, "from a beginning through a middle to an end" (Halliday & Martin, 1993, p. 18). This was underpinned in this study by expanding the logo-genetic evolution to subsume the progression of curriculum ecology which unfolds; that is, from a beginning of a syllabus and curriculum guideline, through a middle of textbooks to an end of examination paper.

Re-instantiation as the key underlying linguistic theory modelling recontextualisation, suggests "the concept of instantiation provides a rationale for the methodology itself" (Hood, 2008, p. 353). Re-instantiation is a model that was proposed to underline "what is happening in the instance, and at the same time an enrichment of the theory to account for different contexts of use" (Hood, 2008, p. 353). In the present study, instantiation accounts for how Pythagoras' Theorem has been presented differently in different educational contexts.

Studies focusing on instantiation (Martin, 2007; Hood, 2008; Caple, 2009; Zhao, 2012; Painter, et. al, 2013) have proposed "commitment" as the theoretical terminology that underlines "degree of meaning potential instantiated in one instance or another" (Hood, 2008, p. 356). In this section, how "commitment" accounts for the instantiation of meaning potential in multi-semiotically constructed instances is revisited with a focus on ideational meaning in particular. Movements in the commitment of ideational meaning are theorized for the purposes of capturing the "serial re-instantiation" (Hood, 2008, p. 356) which is about to emerge from the selected data when the knowledge (Pythagoras' Theorem) is instantiated.

4.7.2 Re-instantiation and commitment

Originating from Halliday's (Halliday, 1991; Halliday & Matthiessen, 2004) work on instantiation, recent development in the work on re-instantiation has been formulated by Martin and his colleagues (e.g. Martin, 2006 and 2008; Hood, 2008; Caple, 2009; Chang, 2011; Painter et al., 2013).

In terms of the semiotic systems involved in re-instantiation and commitment, the work in this field derives from the analysis between different linguistic texts (Martin, 2007; Hood, 2008) and later develops to include other semiotic systems such as visual images in children's picture books (Painter, et al., 2013), and forms and diagrams in physics textbooks (Zhao, 2012). The development of the work on re-instantiation and commitment paved the way for a wider-coverage of research domains that these theoretical underpinnings could include.

Drawing on the research dedicated to the current trends in re-instantiation and commitment, two theoretical considerations implied in previous studies were proposed. The first theoretical consideration taken into account in this study was that re-instantiation could be applied both within a text, such as between different instances of Pythagoras' Theorem in one textbook, and between different physically remote texts, such as between the instances of Pythagoras' Theorem in the syllabus and the instances in a textbook. The other theoretical consideration taken into account in this study is re-instantiation as a multi-semiotic phenomenon, corresponding with the research paradigm outlined in this chapter, describing mathematical discourse as multisemiotic in nature where meaning-making resources such as mathematical symbolism and visual images co-exist with verbal language to construe meaning.

4.8 Adapted model of commitment of ideational meaning in mathematical discourse

This study is not going to directly impose the frameworks of Hood (2008) and Painter et al. (2013) into the analysis. Given the semiotic complexity of mathematics, an adapted model is proposed, essentially synergizing the works of Hood (2008), Painter et al. (2013) and O'Halloran (2007b) to suggest how ideational meaning is committed in mathematical discourse.

Sections 4.8.1 to 4.8.3 provide the procedure necessary to understand the research data in the present study. Section 4.8.4 provides a blueprint of analysis.

4.8.1 Stage 1: "Juxtaposition and Spatiality": the relationship different components within one pedagogic item

As has been noted in Section 4.4, in the present study I adopt a bottom-up approach and start with a pedagogic item as the smallest unit of analysis. The clear cuts made within one pedagogic item are textually informed with the help of "Juxtaposition and Spatiality" in the work of O'Halloran (2007b, pp. 92–93) on the compositional arrangements in mathematics. Theoretically, with the help of "line spacing and centering" (O'Halloran, 2007b, p. 92) different components within one pedagogic item are separated. For example, Figure 4.3 is the pedagogic item taken from Wong and Wong (2009). For the convenience of elaboration, Figure 4.3 is revisited here and is re-numbered as Figure 4.12.

In a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of two adjacent sides.

i.e. In ΔABC, if $\angle C=90°$,

then $a^2+b^2=c^2$.

(Abbreviation: Pyth. theorem)

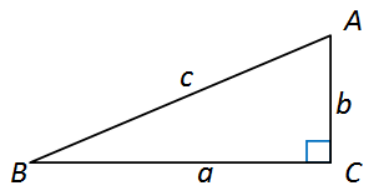


Figure 4.12: Instance of Pythagoras' Theorem selected from Wong & Wong (2009, p. 103)

This instance could be separated into four different components: 1. a lengthy description, 2. a symbolic equation, 3. a technical term and 4. a visual triangle based on the textual layout.

Textually speaking, these components are clearly differentiated from one another, drawing on O'Halloran's "Juxtaposition and Spatiality" (2007b, p. 92 & p. 93). The origin of O'Halloran's "Juxtaposition and Spatiality" can be traced from the work by Kress and van Leeuwen (2006) on visual segmentation, and O'Toole's (1994) work on visual display. This study utilizes the compositional arrangements informed by "Juxtaposition and Spatiality" (O'Halloran, 2007b) in the first place. It is argued that with the help of "Juxtaposition and Spatiality", discernible cuts between different pedagogic items in mathematical discourse could be identified.

In terms of the ideational commitment, each component commits a certain degree of ideational meaning. Theoretical considerations behind the ideational commitment are taken from the work of Painter et al. (2013) on the different commitments of ideational meaning by different components within a bimodal text. For example, each component in Figure 4.3 commits a certain degree of Pythagoras' Theorem.

4.8.2 Stage 2: Identify Semiotic Adoption (O'Halloran, 2007b, p. 92) in each pedagogic item

Semiotic adoption indicates the phenomenon where "system choices from one semiotic resource are incorporated as system choices within another semiotic system" (O'Halloran, 2007b, p. 92). A typical instance of this incorporation of system choices is indicated through the process where symbolic resources have been encoded with linguistic functions and act like experiential features such as participants, circumstances and processes in linguistic statements. The encoding of grammatical functionalities from language into symbolism indicates the process where the grammars of mathematical symbolism "interlock in ways so that selections are almost interchangeable" (O'Halloran, 2005, p. 174). In linguistic statements, encoded symbolic elements will therefore embrace the linguistic features (O'Halloran, 2005).

Semiotic adoption extended the experiential meta-function where the elements in the Transitivity system could include mathematical symbolism as participants, processes and circumstances. Recontextualisation gives rise to semiotic adoption with which experiential functionalities have been relocated from verbal language and delocated into mathematical symbolism.

For example, "+", "-", "×" and "÷" as mathematical symbolisms which recontextualise the operative processes "plus", "minus", "multiply" and "divide" respectively have now become the default expressions in mathematics.

4.8.3 Stage 3: Identify Semiotic Mixing (O'Halloran, 2007b, p. 92) in each pedagogic item

Semiotic mixing describes a situation where different semiotic resources co-occur. Semiotic mixing is designed to explain visual images in mathematical discourse in particular. Unlike classic visual images such as the work introduced by Painter et al. (2013) where images in children picture books could appear as a mono-modal artefact with the involvement of picture only, mathematical visual images are always multisemiotic where linguistic, symbolic and visual elements cooperate. Their co-occurrence gives rise to the emergence of mathematical visual images such as tables, graphs and figures.

One mathematical visual image seldom appears alone. It is recontextualised from other pedagogic items such as a linguistic statement, question, and/or theorem, whose experiential features such as participants, processes and circumstances have been recontextualised as a visual image. Chapter Five will discuss how visual images recontextualise other components within a pedagogic item.

4.8.4 A synergized model and the blueprint of the analysis

A model specific to mathematical discourse summarizes the steps above. Since one typical pedagogic item (such as in Figure 4.3), is composed of different components, to understand how Pythagoras' Theorem is instantiated is actually to understand the level of ideational meaning committed by different components. The commitments of ideational meaning in different components might not be the same. Changes could occur.

Regarding the blueprint of analysis, after digitalizing the printed version of texts into manageable research data in the present study, a series of step is undertaken to investigate research data. Samples of research are displayed in Section 4.9. The initial step of analysis is to lay clear textual cuts within one pedagogic item based on "Juxtaposition and Spatiality" as informed by O'Halloran (2007b). Segmented parts within a pedagogic item are termed components, relying on Cheong's (2004) work on the ideational meaning distribution in multimodal posters. The second step is to identify the semiotic nature of the pedagogic item in order to understand whether there are incidents of semiotic adoption where the recontextualisation between mathematical symbolism and verbal language is found. The third step is to identify whether semiotic mixing could be identified in the pedagogic item. This step is specifically designed for understanding the multi-semiotic phenomenon when mathematical visual images are aligned and juxtaposed in line with components such as linguistic statements and symbolic equations within a pedagogic item. After the identification of the semiotic situation for each pedagogic item, different components within the pedagogic item are analysed through an investigation of how the ideational meaning is committed in each component. Different ideational meanings committed by different components will be discerned in order to account for the overall ideational meaning potential committed in the pedagogic item. The commitment of ideational meaning is dependent on how the knowledge structure of Pythagoras' Theorem has been instantiated through an investigation of the lexical relationship, the recontextualisation relationship between mathematical symbolism and verbal language, and the inter-modal relationship between different components.

4.9 Introducing the data sample

In this section, samples of research data are introduced with a focus on how to interpret the smallest unit of analysis; that is, a pedagogic item from which all the analysis arises. Three typical types of research data are introduced in the subsections: Section 4.9.1 provides a table of mathematical concepts whose semiotic construction is composed through encoding the compositional relationship into tabular format. As for the compositional relationship considered in this study, I draw on Halliday and Martin's (1993) explanation of "classification diagram" where the complex compositional relationship addresses the relationship within a structure as "part, sub-parts, sub-sub-parts and so on" (p. 193), indicating a cline of subordination. Section 4.9.2 provides a linguistic statement in which semiotic complex in terms of the recontextualisation between verbal language and mathematical symbolism has been identified. Section 4.9.3 provides an example of semiotic adoption whose textual layout could be labelled as four components including a linguistic description, a symbolic equation, a technical term and a geometric visual image. Each of the three samples represents one typical example of a pedagogic item where the knowledge of Pythagoras' Theorem has been represented. They are instances of representation of Pythagoras' Theorem identified in pedagogic discourses that are not used in classroom settings (Billett & Choy, 2012, p. 158) in the curriculum ecology.

4.9.1 A table is a pedagogic item

The first example is taken from the Syllabus (EDB, 1999, p. 13), and is displayed in Table 4.3.

Table 4.3: Sample of data: A table is a pedagogic item (EDB, 1999, p. 13)

Measures, Shape and Space Dimension	
Key Stage 3 (S1-S3)	**Key Stage 4 (S4-S5)**
Measures in 2-Dimensional (2D) and 3-Dimensional (3D) figure	
• Estimation in Measurement (6) • Simple Idea of Areas and Volumes (15) • More about Areas and Volumes (18)	
Learning Geometry through an Intuitive Approach	
• Introduction to Geometry (10) • Transformation and Symmetry (6) • Congruence and Similarity (14) • Angles Related with Lines and Rectilinear Figures (18) • More about 3-D Figures (6)	• Qualitative Treatment of Locus (6)
Learning Geometry through a Deductive Approach	
• Simple Introduction to Deductive Geometry (27) • Pythagoras' Theorem (8) • Quadrilaterals (15)	• Basic Properties of Circles (39)
Learning Geometry through an Analytic Approach	
• Introduction to Coordinates (9) • Coordinates Geometry of Straight Lines (12)	• Coordinate Treatment of Simple Locus Problems (14)
Trigonometry	
• Trigonometric Ratios and Using Trigonometry (26)	• More about Trigonometry (29)

Note: Numbers in parentheses denote estimated time ratio for the unit.

This table originally appeared on Page 13 of the Syllabus (EDB, 1999, p. 13) elaborating one dimension: measure, shape and space. A tabular representation is used to elaborate the internal characteristics within this dimension. This table is an artefact standing as one pedagogic item, indicating the compositional relationship between different mathematical concepts. It demonstrates key mathematical concepts within the dimension of measure, shape and space. This dimension could be sub-categorized into five different modules with reference to their own subject knowledge. Table 4.3 displays the clear textual layout of the pedagogic item and spacing between different mathematical concepts with the help of bullet point illustration, lines and space in it. Visual image in the form of the table format is mixed with linguistic resources (wording) and symbolism (bullet point "•")[5]. The overall functionality of the table is to display the "compositional relations" (O'Halloran, 2000, p. 377) between different mathematical concepts. The compositional relation has been encoded into the tabular format.

This sample of research data indicates a typical example of the combination of linguistic resources, mathematical symbols and visual images in a way that the compositional relationship between different mathematical concepts is transcended through textual organisation rather than through verbal language.

4.9.2 A statement

The second example is taken from the Curriculum Guideline (HKEAA, 2007) which is largely composed of linguistic resources with incidents of semiotic adoption.

In the original text (HKEAA, 2007, pp. 113–114), one exemplar displays how "the properties of scalar product of vectors" could be introduced in a classroom setting. This exemplar indicates an embedded relationship between it and its projector with punctuation ":" (colon) indicating the projector, and a large rectangle spanning two pages, indicating the projected parts. This exemplar is an independent pedagogic item in which Pythagoras' Theorem is instantiated. Text Box 4.1 is an adapted version of this pedagogic item, modifying the original format into one single rectangle, and maintaining the original semiotic resources such as bold font and its formal typeface.

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[5] In mathematics, bullet point "•" has two functions: one for textual function and the other for experiential function. As for textual function, for example in Table 4.3, bullet point "•" is adopted to lay clear-cuts between different mathematical concepts. As for experiential function, which will be discussed in Section 4.9.2, bullet point "•" symbolises the mathematical process: multiple.

Teaching one of the properties of the scalar product of vectors using the direct instruction, the inquiry and the co-construction approaches

Teachers may integrate various teaching approaches and classroom practices to introduce the properties of the scalar product of vectors so that the lessons can be more vivid and pleasurable. In this example, teaching one of the properties of the scalar product of vectors, $\mathbf{|a\text{-}b|^2 = |a|^2 + |b|^2 - 2\,(a \bullet b)}$, is used as an illustration.

In previous lessons, the teacher has taught the concepts of magnitudes of vectors and the scalar product of vectors using **direct instruction.** In this lesson, the students are divided into small groups to promote discussion, and the groups are asked to **explore** the geometrical meaning of the property. Here, the **inquiry approach** is adopted, with students having to carry out **investigations** with the newly acquired knowledge related to vectors. During the exploration, the groups may interpret the geometrical meaning differently. Some may consider one of the vectors to be a zero vector and get the above property; but others may relate it to the Pythagoras' Theorem by constructing two perpendicular vectors a and b with the same initial point. Hence, the hypotenuse is $|a\text{-}b|$ and $a \bullet b = 0$ and the result is then immediate. If some groups arrive at this conclusion, the teacher should guide them to discover that their interpretation is only valid for special cases. However, the geometrical meaning of this property is related to the cosine formula learned in the Compulsory Part. If some groups can find that the property is the vector version of the cosine formula, they can be invited to explain how they arrived at this geometrical meaning. If none of the groups can arrive at the actual meaning, the teacher may guide them to find it out by giving prompts. Some well-constructed prompts (or scaffolds), such as asking them to draw various types of triangles and find clues to connect $|a\text{-}b|$, $a \bullet b$, $|a|$ and $|b|$ with triangles drawn, may be provided. The co-construction approach is adopted here.

After understanding the geometrical meaning, the result can be derived by applying the cosine formula learned in the Compulsory Part. The groups are further asked to explore alternative proofs. Here, the inquiry approach is employed. The groups may not think of proving this property with $|x|^2 = x \bullet x$ directly. The teacher may give some hints to guide them. In this case, the teacher and the students are co-constructing knowledge. If the students still cannot prove this property, the teacher can demonstrate the proof on the board using the direct instruction approach. Whatever methods the students use, they are invited to explain their proofs to the class. During the explanation, the teacher and student may raise questions and query the reasoning.

Text Box 4.1: Second example, composed of linguistic resources and mathematical symbolism

The semiotic resources utilized in Text Box 4.1 are largely composed of linguistic resources, with two symbolic equations: "$|a-b|^2 = |a|^2 + |b|^2 - 2\ (a \bullet b)$" and "$|x|^2 = x \bullet x$" inserted. From a semiotic complex perspective, Text Box 4.1 is an example of semiotic adoption with mathematical symbolism encoding the experiential meaning potential and acting like participants, processes and circumstances in composing the mathematical statement such as has been presented in Text Box 4.1. To be more explicit, "||" which stands for "value" and "^{2}" which stands for square act as participants, while "=" which stands for "equate", "-" which stands for "minus" and "•" which stands for "multiple" act like operative processes in both equations.

4.9.3 An example from a textbook

Figure 4.12a is an instance of Pythagoras' Theorem, taken from Wong and Wong (2009, p. 103).

In a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of two adjacent sides.

i.e. In ΔABC, if $\angle C=90°$,

then $a^2+b^2=c^2$.

(Abbreviation: Pyth. theorem)

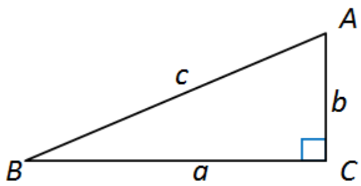


Figure 4.12a: Instance of Pythagoras' Theorem selected from Wong & Wong (2009, p. 103)

In terms of the semiotic resources included, this is a typical example found in mathematical discourse, where verbal language, mathematical symbolism and visual images collaborate to make meaning.

In terms of the mixing of semiotic resources, its geometric visual image is constructed from the following semiotic recourse: a geometric diagram as a triangle, three capitalized letters: A, B and C, three lowercase letters: a, b and c and one symbol: " ┌ ". The construction of this visual image, according to O'Halloran (2007b) is a form of "semiotic mixing" (p. 93), which indicates that "linguistic and symbolic elements" synergize with "the visual display of geometric diagram" (O'Halloran, 2007b, p. 93). The synergy as displayed in Figure 4.3 is that three capitalized letters: A, B and C symbolise three vertices, three lowercase letters: a, b and c symbolise three sides and the symbol " ┌" marks the angle as a right triangle. These semiotic resources collaborate with each other to display a geometric visual image.

The components in Figure 4.13a could be divided into the following four components (listed from Component 4.1 to Component 4.4), separately based on their independent textual layouts.

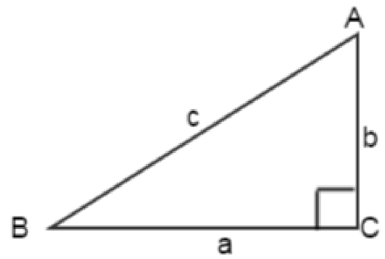
Index	Components
Component 4.1	In a right-angled triangle, the sum of the squares of the two legs is equal to the square of the hypotenuse.
Component 4.2	In ΔABC, if ∠C=90°, then $a^2+b^2=c^2$
Component 4.3	Reference: Pyth. theorem
Component 4.4	

Figure 4.12b: Instance of Pythagoras' Theorem (Wong & Wong, 2009, p. 103)

The commitment of ideational meaning is achieved through two approaches. The first approach is concerned with how the ideational meaning is committed in each of the above components. The second approach is concerned with the ideational meaning committed through the co-patterning relationship synergized by these four components. Being an example of both semiotic mixing and semiotic adoption, this example is a typical instance signifying how mathematical knowledge is instantiated in a pedagogic item. Details regarding how the ideational movement is achieved will be elaborated in following chapters.

4.10 Ethical issue

The research data selected in this study are all from the public domain. The data in this study does not exceed 10% of the overall amount of published data available, thus copyright is not infringed. Because no human participants were involved in this study, ethical concerns for humans did not apply. However, what needs to be mentioned is that the purpose of this research is not intended as a qualitative assessment of the data involved. Rather, this study is intended to underpin the actual practice of knowledge representation in the field of education in Hong Kong. Within the study, I have no invested interests because what I am doing is observation rather than evaluation.

4.11 Summary

This chapter discusses how this study is to proceed. An SFMDA approach is involved to account for the discursive relationship between knowledge and representation, emphasizing that knowledge and representation of knowledge are two separate items. The overview of research data has been based on a curriculum ecology model, highlighting the potential relationship between the data. This potential relationship is argued as a recontextualisation. Moving to a narrower focus of the research data, Pythagoras' Theorem has been highlighted as the focus.

The scope of recontextualisation has been enlarged to account for both the relay between different pedagogic discourses in the curriculum and the knowledge flow within each pedagogic item. The enlargement of the application of recontextualisation is based on a detailed pilot study of one pedagogic item. A blueprint of the analytical steps is provided. Samples of data are also provided.

The ethical issue in this study is resolved by acknowledging that the research data collected is all in the public domain and as such, its use will not result in copyright infringement, as no more than 10% of the original source data has been included.

Chapter Five
Specification of Pythagoras' Theorem's knowledge structure, knowledge progression and recontextualisation

5.1 Introduction

In this chapter, research data extracted from four types of pedagogic discourses are analysed. As presented in Chapter Four, these four types of research data are the Syllabus (EDB, 1999), Curriculum Guideline (HKEAA, 2007), Textbook (Wong and Wong, 2007) and Examination Paper (HKEAA, 2012). The reason for that selection was explained in Section 4.9; that is, to reiterate the reason, the research data selected to be analysed belonged to non-classroom pedagogic discourses. Rather than dismiss the importance of classroom pedagogic discourses in mathematics education, this study focuses on that non-classroom pedagogic discourse, thus confirming Bernstein's (1990) argument that "curricula cannot be acquired wholly by time spent at school" (p. 66). The syllabus, the curriculum guideline, the recommended textbook and the Examination Paper are the non-classroom pedagogic discourses identified in the mathematics curriculum at HKDSE level. Pythagoras' Theorem is the focus of this study for three reasons: first, it is one of the most crucial mathematical concepts in human history, second, it is one of the compulsory mathematical concepts required to be studied at the HKDSE level and third, it is most controversial mathematical concept in both western and eastern civilisation. Instances related to Pythagoras' Theorem are extracted from selected non-classroom pedagogic discourses.

In Section 5.2, I consider instances related to Pythagoras' Theorem extracted from the Syllabus (EDB, 1999) analysing two tables and one flowchart. The analysis of these instances is to examine the relationship between Pythagoras' Theorem and other mathematical concepts. The taxonomic relationship between Pythagoras' Theorem and other mathematical concepts is encoded in the tabular format or symbolized in the flowchart rather than through verbal language. In Section 5.3, I consider instances relating to Pythagoras' Theorem extracted from the Curriculum Guideline (HKEAA, 2007). In the curriculum guideline, Pythagoras' Theorem is represented as a technical term. The taxonomic relationship between Pythagoras' Theorem and other mathematical concepts is achieved with the assistance of verbal language. In Section 5.4, I consider the representation of Pythagoras' Theorem in the mathematics examination paper (HKEAA, 2012). Based on the textual structure of this instance, three different components are identified. They comprise one linguistic statement, one visual image and one checklist in which four possible answers are serialised as A, B, C and D. Each of these components is analysed independently following the blueprint for analysis outlined in Section 4.8. The instantiation of Pythagoras' Theorem in each component is examined from the perspective of ideational commitment. In Section 5.5, I consider the representation of Pythagoras' Theorem in the mathematics textbook based on one selected instance taken from Wong and Wong (2007). Specific to this instance, its textual structure is divided into four different components: one linguistic description, one symbolic equation, one visual image and one technical term. Each of these components is analysed independently following the blueprint for analysis outlined in Section 4.8. The instantiation of Pythagoras' Theorem in each component is examined from the perspective of ideational commitment. In Section 5.5, I also argue that this selected instance is representative for the representation of Pythagoras' Theorem in all qualified mathematics textbooks recommended by the EDB (2007). The generic structure identified in the selected instance (Wong and Wong, 2007) could be repeatedly identified in other textbook instances. This chapter is concluded in Section 5.6.

5.2 Ideational movement in Syllabus: Instances of the representation of Pythagoras' Theorem in the Syllabus

Instances extracted from the Syllabus (EDB, 1999) are analysed in this study. The knowledge structure proposed by Bernstein (2000) is investigated with reference to how Pythagoras' Theorem as one mathematical concept that has been presented in the Syllabus (EDB, 1999). From a textual perspective, the representation of Pythagoras' Theorem in the syllabus utilises a table and a flowchart. Both the table and flowchart formats connect items diagrammatically, addressing the "abstract (inter)textual relations" (Thibault, 1990, p. 134) between these items. As considered by Kress and van Leeuwen (2006, p. 84), the tabular format in which the taxonomic relationship is embedded, and the flowchart format in presenting the knowledge structure, consider the relationship between distinctive items differently. The table taxonomy "represents the world in terms of a hierarchical order" whose main concern is "the ranking for phenomena" (Kress & van Leeuwen, 2006, p. 84). The flowchart describes "the world in terms of an actively pursued process with a clear beginning and an end", with a progression that is "sequential" and "goal-oriented" (Kress & van Leeuwen, 2006, p. 84). The difference between table taxonomy and flowchart in representing the knowledge structure is that flowchart outlines "a map of transactional relationship" (Mohan, 1986, p. 61) between different parts, address the matter concerned here to "be seen as a whole" (Mohan, 1986, p. 61) however it represents "choice very simply and crudely" (Mohan , 1986, p. 63). As for the detailed structure, more detailed thinking and references are required. With respect to table taxonomy, is that conveys "the structure of classification to learners" (Mohan, 1986, p. 6). Compared with flowchart, table taxonomy addresses the detailed internal structure which cannot entirely be conveyed by flowchart for particular items. Therefore, flowchart and table taxonomy always complement each other. This complimentary relationship is identified in the discussion of representations of Pythagoras' Theorem and other mathematical concepts in the Syllabus.

Items in the syllabus referred to different mathematical concepts; and Pythagoras' Theorem is one of them. The diagrammatical relationships between different mathematical concepts address the textual relations. The analysis in this section attempts to bridge the textual relationship exemplified in both tabular and flowchart formats, with the knowledge structure existing at the conceptual level (see Table 5.1.), exploring the conversion between textual layout and conceptual relationship. Textual layout implies the relationship between mathematical concepts at a semiotic level, while the conceptual relationship indicates the relationship between different mathematical concepts at the conceptual level.

Three instances regarding the representation of Pythagoras' Theorem in the Syllabus (EDB, 1999) are introduced. Each instance is discussed separately to explore from a linguistic perspective with reference to the achievement of a taxonomic relationship between different mathematical concepts.

The comparison between these three instances is examined in relation to their different commitments of ideational meaning. A synthesis summarising how Pythagoras' Theorem is represented in Syllabus (EDB, 1999) is also provided, suggesting that a diagrammatical representation of mathematical concepts prevails in the syllabus. This manner of representation is determined by the social function of the Syllabus (EDB, 1999).

5.2.1 First instance: Pythagoras' Theorem in the Syllabus

The first instance of Pythagoras' Theorem in the Syllabus (EDB, 1999, p. 13) is replicated in the following Table 5.1, preserving its original semiotic resources and textual features. The original format has been preserved, such as in the use of bold letters (e.g. **Measures in 2-Dimensional (2D) and 3-Dimensional (3D) figure**), initial capitalization (e.g. Estimation in Measurement), the use of bullet points (e.g. a bullet point "•" precedes every mathematical concept) and the colour of each cell (e.g. the use of the colour "grey" for two cells and the use of the colour "white" for the rest of the cells).

Table 5.1: First instance where Pythagoras' Theorem is represented in the Syllabus

Measures, Shape and Space Dimension	
Key Stage 3 (S1-S3)	**Key Stage 4 (S4-S5)**
Measures in 2-Dimensional (2D) and 3-Dimensional (3D) figure	
• Estimation in Measurement (6) • Simple Idea of Areas and Volumes (15) • More about Areas and Volumes (18)	
Learning Geometry through an Intuitive Approach	
• Introduction to Geometry (10) • Transformation and Symmetry (6) • Congruence and Similarity (14) • Angles Related with Lines and Rectilinear Figures (18) • More about 3-D Figures (6)	• Qualitative Treatment of Locus (6)
Learning Geometry through a Deductive Approach	
• Simple Introduction to Deductive Geometry (27) • Pythagoras' Theorem (8) • Quadrilaterals (15)	• Basic Properties of Circles (39)
Learning Geometry through an Analytic Approach	
• Introduction to Coordinates (9) • Coordinates Geometry of Straight Lines (12)	• Coordinate Treatment of Simple Locus Problems (14)
Trigonometry	
• Trigonometric Ratios and Using Trigonometry (26)	• More about Trigonometry (29)

Note: Numbers in parentheses denote estimated time ratio for the unit.

From a top-down reading path, Table 5.1 starts with a title “Measures, Shape and Space Dimension”, followed by a table and ending with a footnote: “Note: The number in the bracket denotes the estimated time ratio for the unit” standing for the estimated time of teaching. Both the title and the footnote utilise linguistic resources as their way of constructing information. The selection of bold type for the title and the footnote is primarily chosen from an interpersonal perspective, because bold type is one of the “emphatic devices” (i.e. italics, bold type, underlining) (Kress & van Leeuwen, 2006, p. 204) which are deliberately selected by the text writer (in this instance, EDB, 1999) to address that titles and the footnote are ideationally significant representing more central nodes in Table 5.1.

In terms of the semiotic resources used in Table 5.1, its textual layout is ostensibly organised in tabular form. This table is a multi-semiotic artefact representing one pedagogic item. By stating that this table is a multi-semiotic artefact, I draw on Bateman’s (2008) definition of a multimodal artefact that, as defined by him, refers to the situation that “a variety of visually-based modes are deployed simultaneously in order to fulfil a coordinated collection of interwoven communicative goals” (p. 1). In this study, the multimodal artefact defined by Bateman (2008) is re-termed a multi-semiotic artefact for two reasons. First, multi-mode and its adjectival form multimodal in SFL refer to multiple channels of communication. One interpretation of multi-mode is to relate it to different “sensory channels” such as “visual, auditory, tactile, olfactory and gustative sign” (Stöckl, p. 2004, 11) channels. These different sensory channels could be categorised as different “types of contact” (Matthiessen, 2009, p. 24), such as to see, to hear, to touch, to smell and to taste. The other interpretation of multi-mode is to suggest that meaning-making resources are composed of more than just verbal language alone. Resources such as “image, gaze, gesture, movement, music, speech and sound-effect” (Kress and Jewitt, 2003, p. 1) could all be treated as multimodal resources that have their potential to make meaning.

Since the nature of research data in this study is printed data, the study focuses on the visual channel. In order to avoid confusion, the term multimodal is replaced with multisemiotic, indicating that semiotic resources other than verbal language have the potential to make meaning. The multi-semiotic nature of the research data echoes with the multi-semiotic nature of mathematical discourse addressed in Section 3.3. Mode in Bateman's (2008) definition is also replaced with semiotic resource, a term giving rise to the inclusion of mathematical semiotic resources, such as mathematical symbolism and visual image, into the discussion.

Therefore, for the purpose of this present study, Bateman's (2008, p. 1) definition of a multi-modal artefact has been paraphrased as "a variety of visually-based semiotic resources are deployed simultaneously in order to fulfil a coordinated collection of interwoven communicative goals". This paraphrased version fits with the nature of written mathematics discourses explored in this present study. This paraphrased version is proposed to account for the instances of Pythagoras' Theorem concerned in this present study. By saying that visually based semiotic resources are "deployed simultaneously" (Bateman, 2008, p. 1), I address the situation where different semiotic resources work together rather than unfold in sequential order. By saying, that these semiotic resources are "orchestrated" (Bateman, 2008, p. 1), I suggest that the semiotic resources in one multi-semiotic artefact such as the one in Table 5.1, work together to make meaning. This confirms O'Halloran's (2005) elaboration that in mathematical discourses, "the systems of meaning for language, symbolism and visual images are integrated" (p. 158). By referring to "an orchestrated collection of interwoven communicative goals" (Bateman, 2008, p. 1), two properties are assigned to this nominal group. First, each semiotic resource identified in one multi-semiotic artefact has its unique way of making meaning, confirming the intra-semiotic nature of mathematical semiotic resources (O'Halloran, 2005). Second, these semiotic resources interweave with each other to achieve communication goals. This second property confirms the inter-semiotic nature of mathematical semiotic resources again (O'Halloran, 2005).

Therefore, in the present study, the property of one multi-semiotic artefact entails the following aspects, namely visual channel and multiple semiotic resources. Each of the semiotic resources has its own functionality while at the same time these semiotic resources interact with each other to express the totality of meaning.

As for the instance presented in Table 5.1, a concept table is introduced with 26 nominal groups, generating an example of semiotic mixing (O'Halloran, 2007b) where the tabular format is mixed with other semiotic resources such as linguistic resources of wording, and graphical resources such as: bullet points "•", brackets and numbers. The overall functionality of the table is to display the "compositional relations" (O'Halloran, 2000, p. 377) between different items, because "different compositional arrangements" will "allow the realization of different textual meanings" (Kress & van Leeuwen, 2006, p. 43).

The discussion starts with bold type. Bold type as the emphatic device is also found in the table as five nominal group heads: Measures in 2-Dimensional (2D) and 3-Dimensional (3D) figure, Learning Geometry through an Intuitive Approach, Learning Geometry through a Deductive Approach, Learning Geometry through an Analytic Approach, and Trigonometry, are all in bold type. From an interpersonal perspective, this selection of bold type indicates the EDB preference in focusing on and highlighting what they believe is important information in Table 5.1. From a textual perspective, the items selected to be in bold type have one quality in common: they are all "Macro-themes" (Martin, 1992), appearing as headings and titles. As for one heading, "Measures in 2-Dimensional (2D) and 3-Dimensional (3D) figure", it is a departure of information until the next succeeding heading (in this table, the next succeeding heading is "Learning Geometry through an Intuitive Approach"). This textual arrangement of information is recurrently identified as a way of organising different mathematical concepts in the Syllabus (EDB, 1999).

Pythagoras' Theorem in this instance is listed as one of the key learning objectives under the dimension of "Measures, Shape and Space" (EDB, 1999, p. 13) and is rendered in bold type, signifying it is the Macro-theme of this subsection. "Simple Introduction to Deductive Geometry (27)", "Pythagoras' Theorem (8)" and "Quadrilaterals (15)" at Key Stage 3, together with "Basic Properties of Circles (39)" at Key Stage 4, are the information led by "Measures, Shape and Space", exemplifying the content of "Measures, Shape and Space". From a knowledge construction perspective, "Measures, Shape and Space" as one independent dimension has been sub-categorized into five different modules with reference to their own subject knowledge. As articulated by EDB (1999), the organisation of mathematical concepts within one dimension is that they share a "similar nature" (p. 6). As for subsections grouped under one dimension, they are so constructed due to their "inter-relation" (EDB, 1999, p. 8). As for mathematical concepts grouped under one subsection, EDB (1999) also named their relationship as an "inter-relation". In order to avoid confusion between different mathematical concepts, a hierarchy of technical terms is devised. **Measure Shape and Space** is called a dimension, "**Measures in 2-Dimensional (2D) and 3-Dimensional (3D) figures**", "**Learning Geometry through an Intuitive Approach**", "**Learning Geometry through a Deductive Approach**", "**Learning Geometry through an Analytic Approach**", and "**Trigonometry**" are sub-sections and **Pythagoras' Theorem** is one of the key learning objectives.

As for Pythagoras' Theorem, it is co-related with "Simple Introduction to Deductive Geometry", and "Quadrilaterals". These three mathematical concepts are parts of the subsection: "Learning Geometry through a Deductive Approach" based on EDB's (1999, p. 8) understanding.

The EDB's understanding of the relationship between Pythagoras' Theorem and other mathematical concepts is expressed in their overview of the mathematics curriculum (EDB, 1999, pp. 6–16). In terms of the specific knowledge construction of Pythagoras' Theorem, Table 5.1 extracted from EDB (1999, p. 13) is investigated in this study.

From an ideational perspective, nominal groups are identified in Table 5.1. The verbal group that is also an important resource in construing ideational meaning is omitted. The omission of the verbal group does not equate to the omission of the verbal process. Rather, verbal processes are encoded in the tabular format, confirming what Lemke (1998) calls "textual ellipsis" (p. 96). In this instance the "compositional relation" (O'Halloran, 2000, p. 377) between different mathematical concepts is indicated through the table format, indicating "the structure of classification" provided by Mohan (1986, p. 90). In Table 5.1, "visual organisational resources" (Lemke, 1998, p. 96) such as the spacing between different mathematical concepts, the bullet point illustration, different lines and different boxes enable the compositional relation to "be recovered from bare thematic items in the absence of grammatical constructions" (Lemke, 1998, p. 96), such as the absence of the verbal group in this instance.

It must be noted that Table 5.1 is a conceptual table where the relationship between different concepts are presented. The use of conceptual tables in the Syllabus (EDB, 1999, p. 13) encodes only the compositional relationship, fitting into the category of relational processes. In mathematics, there are other tabular forms, such as matrices, where operative processes are encoded. As for the instance in Table 5.1, a compositional relationship is encoded in the table.

As for the relationships between different nominal groups in Table 5.1, their relationships are concerned with "the distribution of the information value" (Unsworth, 2008, p. 3) with reference to the layout of Table 5.1. Within one multi-semiotic artefact, information value relates to "the placement of items in relation to each other" (Ravelli, 2008, p. 21) in this artefact. As for information value conveyed in Table 5.1, mathematical concepts such as "Learning Geometry through a Deductive Approach" are more general than mathematical concepts such as "Pythagoras' Theorem". In saying that one is more general than another, I refer to their distinction in terms of "generality" (Halliday, 2004, p. 64). According to Halliday (2004), "the superordinate category is more general than its hyponyms" (p. 64). In Table 5.1, with respect to the compositional layout, "Learning Geometry through a Deductive Approach" is the superordinate category while "Pythagoras' Theorem" is one of the hyponyms, because visually speaking "Learning Geometry through a Deductive Approach" is placed right above the word box containing "Pythagoras' Theorem". Therefore, "Learning Geometry through a Deductive Approach" is more general than "Pythagoras' Theorem". Their distinction in terms of "generality" (Halliday, 2004, p. 64) is investigated through their taxonomic relationship. The taxonomic relationship was initially proposed to describe the lexical relationship between different lexical items (Martin & Rose, 2003, 2007, 2014; Halliday, 2004). With respect to different levels of "generality" identified in Table 5.1, the taxonomic relationship is extended to cover the lexical relationship between different lexis in the tabular format.

Specific to *Pythagoras' Theorem*, a staged meronymic relationship could be identified: "*Pythagoras' Theorem"* is a part of "*Learning Geometry through a Deductive Approach"* which is in turn a part of the *"Measure, Shape and Space Dimension".* The ideational commitment of Pythagoras' Theorem in this instance is through generalisation (Hood, 2008, p. 357). This generalisation informs a two-stage meronymic relationship, indicating the compositional relationship between Pythagoras' Theorem and other mathematical concepts. This compositional relationship is explicitly indicated through the tabular format rather than through verbal language.

The compositional relationship between different mathematical concepts is parallel to the development of the knowledge structure. According to Bernstein (1999/2000), in the case of hierarchical knowledge construction, the development between different mathematical concepts is that knowledge at higher levels "is more general, more integrating" (p. 163), than knowledge at lower levels since "hierarchical knowledge structures appear to be motivated towards greater and greater integrating propositions" (Bernstein, 1999, p. 162).

In terms of the knowledge structure of Pythagoras' Theorem, the two-stage meronymic relationship indicated by the tabular construction informs a hierarchical knowledge organisation with "Pythagoras' Theorem" being comprehensively integrated by "Learning Geometry through a Deductive Approach" which is in turn comprehensively integrated within "Measure, Shape and Space Dimension". To argue from the construction perspective of hierarchical knowledge, "Measure, Shape and Space Dimension" is more general than "Learning Geometry through a Deductive Approach" which is in turn more general than "Pythagoras' Theorem", confirming the "'integrating' code" of hierarchical knowledge structures.

Bernstein (1999, 2000), proposes a triangle as the model of hierarchical knowledge structure. Wignell (2007) extends Bernstein's (1999, 2000) proposal, and provides a model of warring triangles to suggest the knowledge structure in social science, because although not provided in his writing, Bernstein (1999) also believes that "there is likely to be more than one triangle in a hierarchical knowledge structure" (p. 171). Following Wignell's (2007) warring triangles model, O'Halloran (2007a) suggests that the knowledge structure within mathematics "would look something like a series of triangles which ... have the potential to be in conflict with each other". The model by O'Halloran (2007a, p. 207) is provided in Figure 5.1.

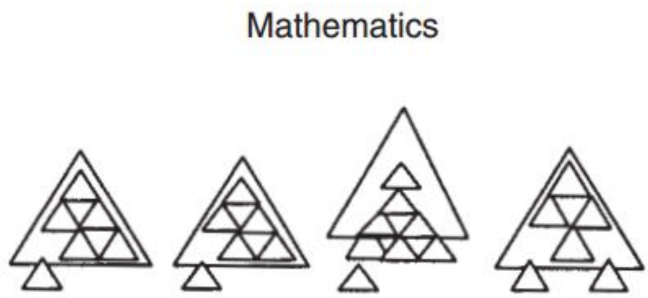


Figure 5.1: O'Halloran's model of mathematical knowledge structure

In Figure 5.1, four discrete triangles stand for the horizontal knowledge structures within mathematics (Bernstein, 1999 & 2000). These horizontal knowledge structures could be identified in the relationship between algebra, geometry and statistics. Within each triangle, smaller triangles are found. According to O'Halloran (2007a), "triangles within triangles … together form an integrated hierarchy of knowledge" (p. 207), confirming the default property of the hierarchical knowledge structure: "integrating" as elaborated by Bernstein (1999, p. 163).

Relying on Bernstein's (1999, 2000) triangle in portraying the hierarchical knowledge structure, Martin (2011, p. 42) proposes a series of overlapping triangles which capture the integration existing within the hierarchical knowledge structure. This model is presented in Figure 5.2.

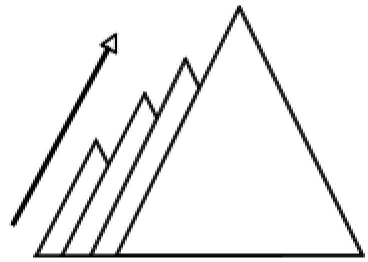


Figure 5.2: Progression of hierarchical knowledge structure

In Figure 5.2, different sizes of triangles stand for different concepts and the overlapping sections acknowledges the existence of integration within the larger triangle, partially integrating the smaller triangles. The arrow indicates the direction of integration, moving from the least integrated to the most integrated. As for what meaning is encoded in the size of the triangle, Bernstein (1999) admits that the triangle "attempts to create very general propositions and theories" (p. 162). The motivation for using a triangle is concerned with "the broadest base and the most powerful apex" (Bernstein, 1999, p. 171) underlined in the triangle.

Therefore, incorporating O'Halloran's (2007a) extension of Bernstein's (1999, 2000) model in mathematics, and also borrowing Martin's (2011) model of overlapping triangles, the hierarchical knowledge structure concerned with Pythagoras' Theorem in Table 5.1 could be portrayed in Figure 5.3 in the form of three overlapping triangles.

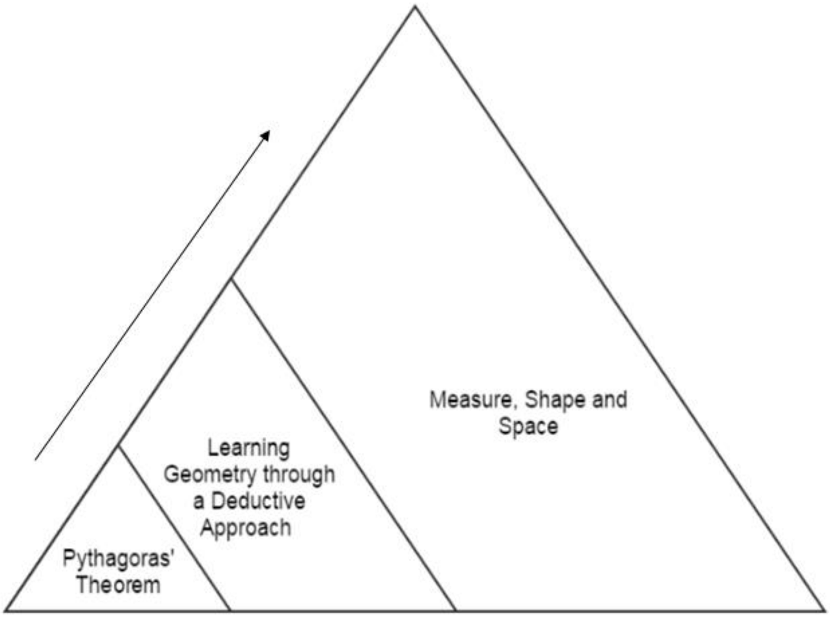


Figure 5.3: Hierarchical knowledge organisation in the first instance

Figure 5.3 is composed of three different triangles: one for "Pythagoras' Theorem", the other for "Learning Geometry through a Deductive Approach" and the third for "Measure, Shape and Space". An arrow indicates the direction of integration. The three triangles overlap each other with Pythagoras' Theorem being comprehensively integrated by "Learning Geometry through a Deductive Approach" which is in turn comprehensively integrated by "Measure, Shape and Space". As indicated by the arrow, the integration starts from Pythagoras' Theorem. As considered by EDB (1999), Pythagoras' Theorem is part of the prerequisite knowledge for "Learning Geometry through a Deductive Approach" which is in turn part of the prerequisite knowledge for "Measure, Shape and Space" from the perspective of hierarchical knowledge construction.

Specific to this instance, with reference to Bernstein's (1999, 2000) hierarchical knowledge structures, Pythagoras' Theorem is integrated by "more integrating and more abstract" (Bernstein, 1999, p. 163) mathematical concepts such as "Learning Geometry through a Deductive Approach". Although termed an "inter-relation" by EDB (1999, p. 8), this integration of knowledge structure is further explained with the assistance of the tabular format in Table 5.1. The compositional relationship embedded within the structure of the table can be used to examine different levels of generalisation (Hood, 2008) between relevant mathematical concepts. In this instance, "Pythagoras' Theorem" is the least general mathematical concept.

5.2.2 Second instance in the syllabus

The second instance of Pythagoras' Theorem is found on Page 23 in the Syllabus (EDB, 1999). The semiotic resources in its original place are preserved here as well. This second instance is presented in Table 5. 2.

Table 5.2: The second instance of Pythagoras' Theorem in the syllabus (EDB, 1999, p. 23)

Unit	Learning Objectives	Suggested time ratio (min.)
Learning Geometry through a Deductive Approach		
Simple Introduction to Deductive Geometry	• develop a deductive approach to study geometric properties through studying the story of Euclid and his book- *Elements* • develop an intuitive idea of deductive reasoning by presenting proofs of geometric problems relating with angles and lines • understand and use the conditions for congruent and similar triangles to perform simple proofs • identify lines in a triangle such as medians, perpendicular bisectors etc. • explore and recognize the relations between the lines of triangles such as the triangle inequality, concurrence of intersecting points of medians etc. • explore and justify the methods of constructing centres of a triangle such as in-centre, circumcentre, orthocentre, centroids etc. • ** prove some properties of the centres of the triangle	27
Pythagoras' Theorem	• recognize and appreciate different proofs of Pythagoras' Theorem including those in Ancient China • recognize the existence of irrational numbers and surds • use Pythagoras' Theorem and its converse to solve problems • appreciate the dynamic element of mathematics knowledge through	8

Unit	Learning Objectives	Suggested time ratio (min.)
	studying the story of the first crisis of mathematics • **investigate and compare the approaches behind in proving Pythagoras' Theorem in different cultures • **explore various methods in finding square root	
Quadrilaterals	• extend the idea of deductive reasoning in handling geometric problems involving quadrilaterals • deduce the properties of various types of quadrilaterals but with focus on parallelograms and special quadrilaterals • perform simple proofs related with parallelograms • understand and use the mid-point and intercept theorems to find unknowns	15

Note: Objectives with asterisk (**) are considered figures of **enrichment topics**. Objectives underlined are considered a **non-foundation** part of the syllabus.

From an overview perspective, the second instance (EDB, 1999, p. 23) is a continuum of the first instance (EDB, 1999, p. 13) because repetition of the same lexical items in both instances is identified.

Table 5.3 outlines the repeated lexical items identified in both instances.

Table 5.3: Lexical repetition identified in EDB, 1999, p. 13 and EDB, 1999, p. 23

Page of the Syllabus	Repeated Lexical items
EDB, 1999, p. 13	• **Learning Geometry through a Deductive Approach** • Simple Introduction to Deductive Geometry • Pythagoras' Theorem • Quadrilaterals
EDB, 1999, p. 23	• **Learning Geometry through a Deductive Approach** • Simple Introduction to Deductive Geometry • Pythagoras' Theorem • Quadrilaterals

It is identified that, in addition to the repetition of linguistic resources in both instances, semiotic resources such as the tabular format, the numbers and the "emphatic device" (Kress & van Leeuwen, 2006, p. 204) such as the bold type used for “**Learning Geometry through a Deductive Approach**” have also been repeated. The repetition existing at both lexical and non-linguistic levels creates the cohesion existing between Tables 5.1 and 5.2.

From the stand point that the relationship between Tables 5.1 and 5.2 could be regarded as two independent texts that are cohesive with each other, the relationship between them is that Table 5.2 is the continuum of Table 5.1 with Table 5.2 adding more delicate layers to Table 5.1.

With regard to the meaning-making process specific to Table 5.2, the methods applied in Table 5.1 could also be incorporated in this section.

Similar to Table 5.1, Table 5.2 is also a multi-semiotic artefact where semiotic resources such as verbal language and tables are deployed. As for the use of “emphatic devices” (Kress & van Leeuwen, 2006, p. 204) such as bold type, italics, underlining and asterisks, ideational meaning, interpersonal meaning and textual meaning are encoded in them. As for the ideational meaning, their marking of central nodes in the network of ideational meaning suggests the important places in creating the table taxonomy. As for interpersonal meaning, bold type such as “**Learning Geometry through a Deductive Approach”** and italics (such as *Elements*) are used to express the attitude held by EDB (1999, p. 23) in terms of the significant information conveyed in these terms. As for textual meaning, both asterisk (**) and <u>underlining</u> are cataphoric derivatives, indicating that information is to be retrieved in the following text (Halliday & Hasan, 1976). The sequence between different information is determined by their sequential order in the Syllabus (EDB, 1999), expressed through their page numbers. In this instance, an asterisk is related to information that is considered an “enrichment topic” (EDB, 1999, p. 23). The content of enrichment topics that are provided on Page 46 of the Syllabus (EDB, 1999) is related to “simple games and real-life activities” (EDB, 1999, p. 46). As for <u>underlines</u>, they are related to information that is considered a “non-foundation part of the syllabus” (EDB, 1999, p. 23), and is elaborated on Page 6 of the Syllabus (EDB, 1999). The foundation part of the Syllabus constitutes “a set of essential concepts and knowledge which all students should strive to learn” (EDB, 1999, p. 6) while the non-foundation part … is designed for students who are "more able in mathematics, more mathematically oriented or need more mathematical knowledge and skills to prepare for their future studies and careers" (EDB, 1999, p. 6). Rather than explaining the distinction between the foundation and non-foundation parts using linguistic resources, the use of underlining provides an economic way to encode meaning to the non-foundation part by underlining, and enabling these underlined exophoric devices to convey meaning outside the confines of Table 5.2.

After identifying the interpersonal and textual meanings conveyed by emphatic devices, the focus of analysis shifts to examining the construction of compositional relationship embedded in the tabular structure of Table 5.2.

As outlined in Section 5.2.1, the compositional relationship between different mathematical concepts is related to the format and function of the table. This nature of the table could be applied in Table 5.2 as well. The relationship between different mathematical concepts is clarified with the assistance of a bullet point illustration, separate lines, boxes and spaces.

Similar to the analysis in Section 5.2.1, a staged meronymic relationship could be identified between *"Learning Geometry through a Deductive Approach"*, *"Pythagoras' Theorem"* and the six subsections of Pythagoras' Theorem. From a taxonomic relationship perspective, the meronymic relationship is indicated through the textual layout of tables, lines boxes and spacing. Therefore, the ideational commitment of Pythagoras' Theorem in this instance is through generalisation (Hood, 2008, p. 353). This generalisation is a two-staged meronymic relationship, indicating the compositional relationship between Pythagoras' Theorem and other mathematical concepts.

As for the relationship between Pythagoras' Theorem and its six subsections, Figure 5.4, adopted from O'Halloran's (2007a, p. 207) model, describes their relationship. Each triangle "△"represents a subsection within Pythagoras' Theorem.

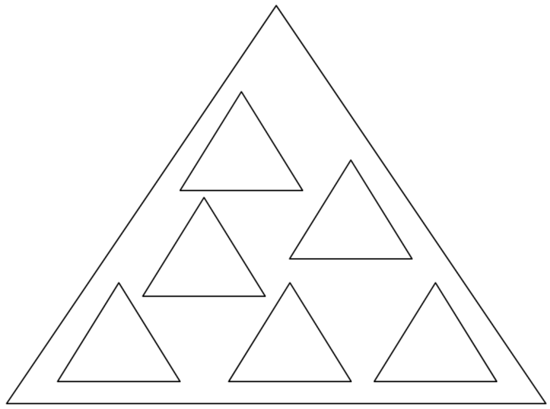


Figure 5.4: Pythagoras' Theorem and its six subsections

To clarify, these six subsections within Pythagoras' Theorem are considered as conflicting with each other. Each of them describes a certain property of Pythagoras' Theorem. The display of these triangles is also general, suggesting their visual arrangements of them does not predict their distinction in terms of the information value. In this example, they are treated only as subsections of Pythagoras' Theorem.

The tabular format is the method applied to indicate compositional relationship. A knowledge structure at the conceptual level converted from the compositional relationship is provided in Figure 5.5. The knowledge structure between different mathematical concepts is inspired by Bernstein's (2000) model of knowledge structure.

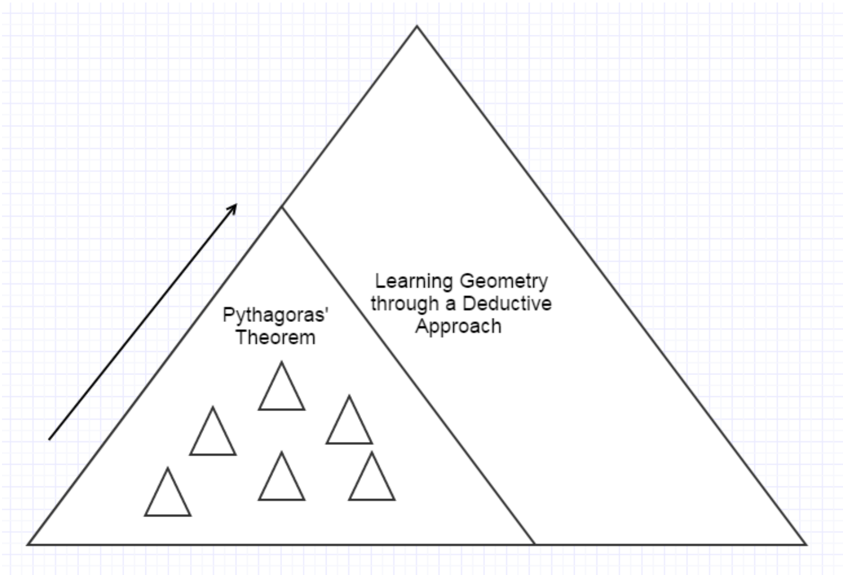


Figure 5.5: Knowledge structure between six subsections within Pythagoras' Theorem, and Pythagoras' Theorem within Learning Geometry through a Deductive Approach

Evidenced through the tabular format available in Table 5.2, this figurative portrayal in Figure 5.5 indicates the hierarchical knowledge construction concerning with the Pythagoras' Theorem. The arrow in Figure 5.5 indicates a direction of integration that starts from the six subsections (six small triangles "△") within Pythagoras' Theorem. The model of conflicting triangles displayed in Figure 5.4 is merged into Figure 5.5 for the purposes of highlighting the compositional relationship in Table 5.2. In Figure 5.5, eight different triangles are identified. The largest triangle stands for "learning geometry through a deductive approach" and a comparatively smaller triangle represents. "Pythagoras' Theorem" is a sub-set of "learning geometry through a deductive approach". Six subsections within Pythagoras' Theorem are represented by six different smaller triangles, each standing for one property within the Theorem. A staged meronymic relationship is identified based on Figure 5.5.

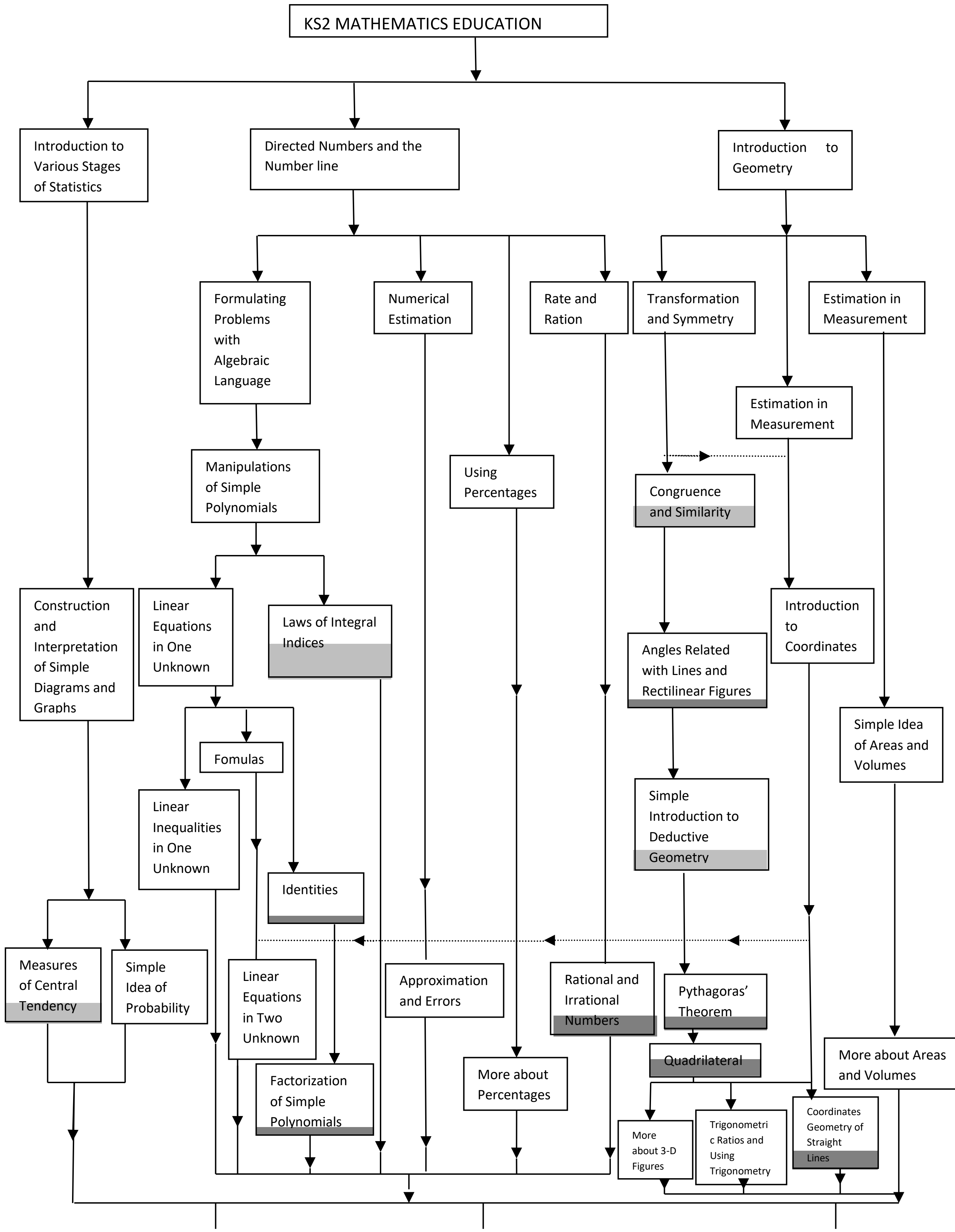
The meronymic relationship indicated through textual layout suggests the hierarchical knowledge construction considered by Bernstein (1999, 2000). This approach is reached based on the evidence provided in both Sections 5.2.1 and 5.2.2. A hierarchical knowledge structure is presented based on the table which is the major semiotic resource applied to construct textual metafunction.

The six subsections within Pythagoras' Theorem are the internal characteristics of Pythagoras' Theorem, forming a meronymic relationship between them within Pythagoras' Theorem. In terms of the internal relationship between these six subsections, their juxtaposition (marked by bullet point "•") and their separate spacing (six individual space positions) open up a potential to incorporate Bernstein's (2000) vertical discourse structure, indicating that they are connected and their connection leads to two possibilities. These possibilities are either horizontal knowledge structure with the existence of incommensurability between them, or a hierarchical knowledge structure where one is absorbed by another.

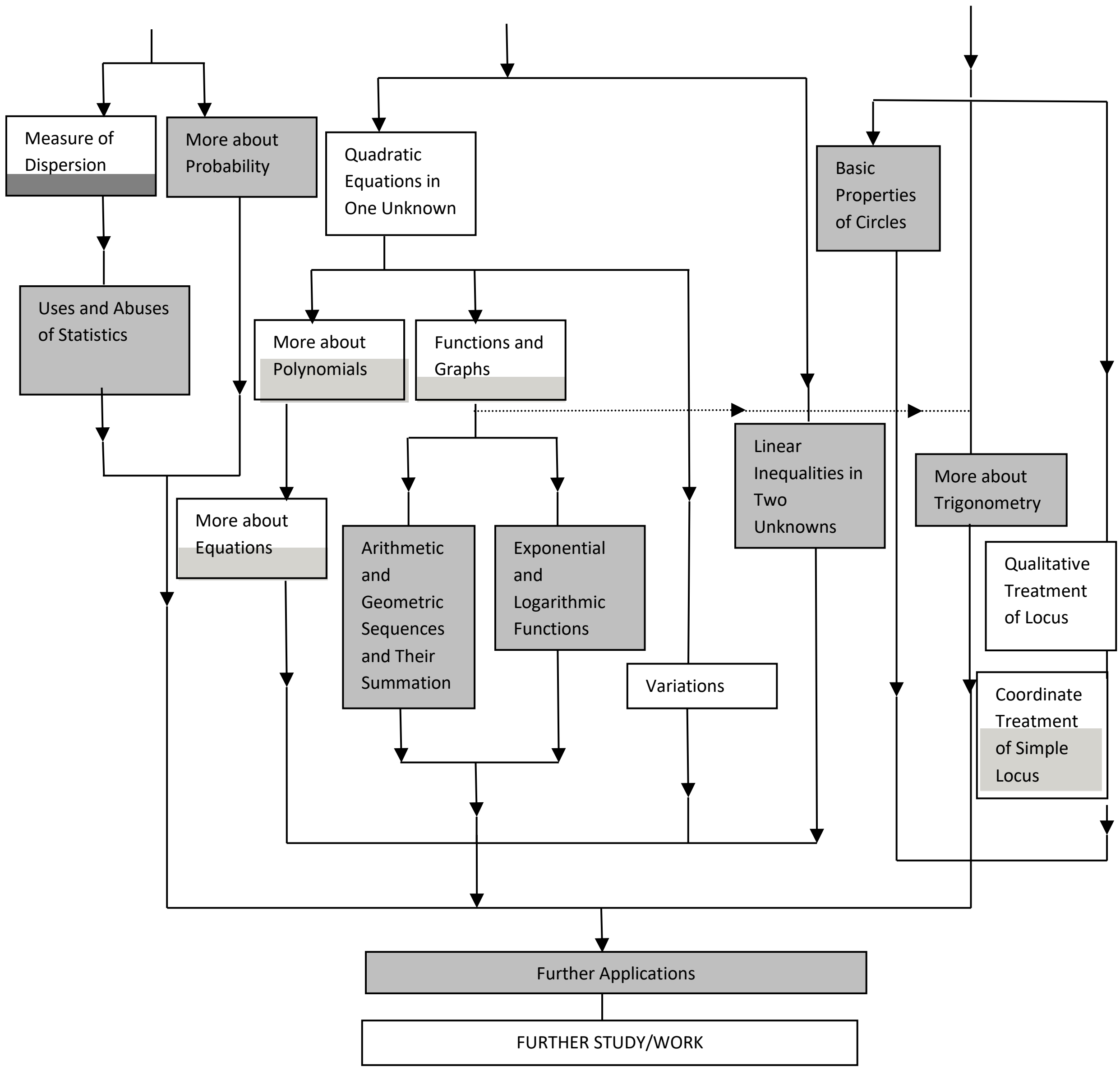
Therefore, the ideational commitment of Pythagoras' Theorem in this instance is achieved through the tabular format, foregrounding the hierarchical knowledge structure scaffold by the meronymic relationship between "*six subsections within Pythagoras' Theorem*", *"Pythagoras' Theorem"* and *"Learning Geometry through a Deductive Approach"*. This instance also commits the possibility of incorporating the vertical knowledge structure between six subsections within Pythagoras' Theorem, opening up the potential for subsuming either the horizontal knowledge structure or the hierarchical knowledge structure.

5.2.3 Pythagoras' Theorem: The flowchart of the network

The network demonstrating key mathematical concepts at the HKDSE level has been provided by EDB (1999) on Page ANNEX III. This diagrammatical representation of mathematics knowledge is replicated in Figure 5.6.


KS2 MATHEMATICS EDUCATION
Introduction to Various Stages of Statistics
Directed Numbers and the Number line
Introduction to Geometry
Formulating Problems with Algebraic Language
Numerical Estimation
Rate and Ration
Transformation and Symmetry
Estimation in Measurement
Estimation in Measurement
Manipulations of Simple Polynomials
Using Percentages
Congruence and Similarity
Construction and Interpretation of Simple Diagrams and Graphs
Linear Equations in One Unknown
Laws of Integral Indices
Angles Related with Lines and Rectilinear Figures
Introduction to Coordinates
Simple Idea of Areas and Volumes
Fomulas
Linear Inequalities in One Unknown
Simple Introduction to Deductive Geometry
Identities
Measures of Central Tendency
Simple Idea of Probability
Linear Equations in Two Unknown
Approximation and Errors
Rational and Irrational Numbers
Pythagoras' Theorem
More about Areas and Volumes
Quadrilateral
Factorization of Simple Polynomials
More about Percentages
More about 3-D Figures
Trigonometric Ratios and Using Trigonometry
Coordinates Geometry of Straight Lines

Continuum of the flowchart



Key: NON-FOUNDATIONS PARTS

Figure 5.6: Flowchart of Learning Units for Secondary School Mathematics Curriculum (adapted from EDB, 1999, p. ANNEX III)

Note: Mathematical knowledge is interrelated both within and across dimensions. It is important to illustrate all links in a flowchart. Strong links between learning units are shown in dotted lines. These lines are just for illustrations and are not meant to be exhaustive. Teachers should exercise their judgment in arranging the sequence of learning units with special attention to the prerequisite knowledge required. For example, students are required to have the prerequisite knowledge in “Introduction to Coordinates” to solve “Linear Equations in Two Unknowns” by graphical methods.

This flowchat in totality is a pedagogic item demonstrating a complete network structure between different mathematical concepts required for learning at the secondary education level of mathematics. Starting from the semiotic mixing perspective, this flowchart is a type of mathematical visual image where lines, spaces, boxes and linguistic resources are arranged to effect the relationship between different mathematical concepts. Drawing from Kress and van Leeuwen’s (2006) description of the function of a flowchart, this flowchart is sequenced with “KS2 MATHEMATICS EDUCATION” as the start and “Further study/work” as the end. The sequential order presented in the flowchart is orgnised as a temporal order, beginning with KS2 (Key Stage Two) level, passing through KS3 (Key Stage Three) level and KS4 (Key Stage Four) level and ending at further study and further work after the completion of KS4 education. The sequential progression identified in the flowchart, confirming the “linear representation” (Kress & van Leeuwen, 2006, p. 84), is reflected in the organisation of the education system for the mathematics curriculum, progressing step by step from KS2, KS3 to KS4.

As for the specific semiotic resources used in Figure 5.6, there are two different types of lines used in the flowchart. Dotted arrows: “┄◄┄” are displayed on the horizontal axis with solid arrows and “ ↓ ” are displayed on the vertical axis. The direction of the arrow indicates the progression of information flow. The use of different arrows is the default way of representing the sequential order in time and space. The shared and default functionalities are to suggest that different mathematical concepts are connected (EDB, 1999, p. ANNEX III). With reference to the specific functionality for each line, dotted arrows "⇠"indicate that two mathematical concepts are linked with the departure of the arrow being the "prerequisite knowledge" for the end of the arrow (EDB, 1999, p. ANNEX III). For example, as indicated in the footnote of Figure 5.6, the EDB believes that the "Introduction to Coordinates" is prerequisite knowledge for "Linear Equations in Two Unknowns" because they are connected by dotted arrows "⇠" with "Introduction to Coordinates" being the point of departure and "Linear Equations in Two Unknowns" being the end-point of their connection.

Solid arrows " ↓ "indicate that mathematical concepts are conceptually linked and address the sequential order with reference to their presence in both the flowchart and the mathematics curriculum. The purpose of this flowchart is to demonstrate how mathematical knowledge could be understood as a connected network perceived by the EDB. It must be noted that from a knowledge construction perspective, only dotted arrows "⇠" have been encoded with the function of "prerequisite knowledge" (EDB, 1999, p. ANNEX III), the subject of a footnote to this flowchart. From a top-down reading path, solid arrows " ↓ " are used to connect two mathematical concepts with the top one being introduced earlier than the lower one. This sequential order at a textual level accounts for the relationship at the knowledge construction level, with the top level being prerequisite knowledge for the lower level. Therefore, the functions of solid arrows " ↓ ", although not explicated in Figure 5.6, could also be categorized as a method for identifying the prerequisite knowledge between different mathematical concepts.

The description of prerequisite knowledge entails the hierarchical knowledge structure in Bernstein (2000), with subsequent knowledge being more integrating than prerequisite knowledge.

Moving from the overarching meaning-making process in Figure 5.6 to the specific place where Pythagoras' Theorem has been represented as the focus of study, issues arise regarding the semiotic resources needed to realise Pythagoras' Theorem and the relationship between Pythagoras' Theorem and other mathematical concepts.

Unlike the default compositional nature of the tabular format explicated in Sections 5.2.1 and 5.2.2, the default nature of the flowchart indicates a direction of progression such as the sequential order in presenting different entities across time and space. The relationship between different entities (in this example, the mathematical concepts) is achieved through encoding the relationship with different arrows. With respect to Pythagoras' Theorem, its relationship with "Simple Introduction to Deductive Geometry" and "Quadrilateral" is considered. With the help of the solid arrow: " ↓ ", "Simple Introduction to Deductive Geometry" is introduced before "Pythagoras' Theorem" which is in turn introduced before "Quadrilateral". From a knowledge construction perspective, "Simple Introduction to Deductive Geometry" is the prerequisite knowledge for "Pythagoras' Theorem" which is in turn the prerequisite knowledge for "Quadrilateral".

The flowchart format in presenting the relationship between different mathematical concepts in this section has assisted in further elaborating the taxonomic relationship identified in Table 5.1. In Table 5.1, the relationship between "Simple Introduction to Deductive Geometry", "Pythagoras' Theorem" and "Quadrilateral" are understood as they are organised as a vertical discourse, without specifying the Hierarchical Knowledge structure outlined in the flowchart in Figure 5.6. With reference to Mohan's (1986) work on table taxonomy and flowchart, both of them are used for the purpose of classification. The flowchart considered in this study views the complete mathematics curriculum "as a whole" (p. 61), from the primary level to the secondary level. Each particular part of that flowchart can be "isolated and worked on" (Mohan, 1986, p. 61). For example, Pythagoras' Theorem as the particular part isolated from the flowchart was worked on. Table 5.1 and Table 5.2 detail the flowchart and "develop greater understanding of fundamental knowledge structures" (Mohan, 1986, p. 61) which has not been achieved due to the nature of flowchart.

Relying on the knowledge construction model provided by Bernstein (2000), Figure 5.7 visualises the hierarchical knowledge structure between "Simple Introduction to Deductive Geometry", "Pythagoras' Theorem" and "Quadrilateral", previously considered in Table 5.1.

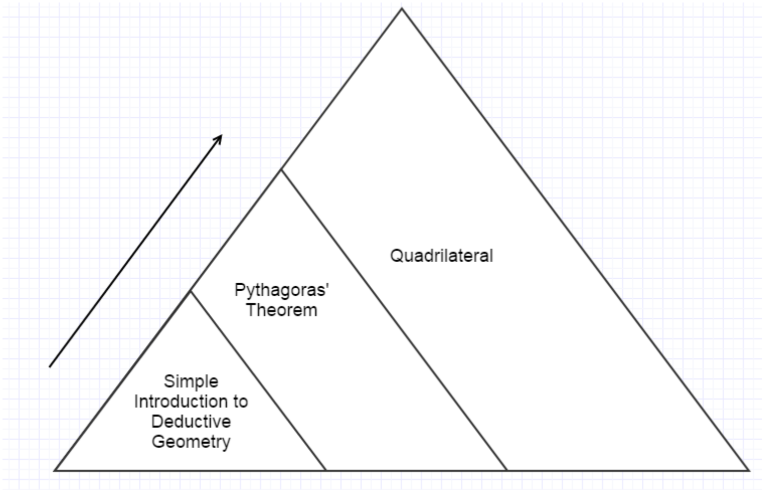


Figure 5.7: Hierarchical knowledge structure between "Simple Introduction to Deductive Geometry", "Pythagoras' Theorem" and "Quadrilateral"

In Figure 5.7, three triangles are identified, one being for the "Simple Introduction to Deductive Approach", the other for "Pythagoras' Theorem" and the third for "Quadrilateral". The arrow indicates the direction of integration, starting from the least integrated level to the most integrated level; that is, "Simple Introduction to Deductive Approach" is integrated by "Pythagoras' Theorem" which is in turn integrated by "Quadrilateral".

The semiotic construction of Pythagoras' Theorem in Figure 5.6 also involves the use of different colours. Figure 5.8 provides the text box specific to Pythagoras' Theorem.

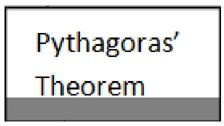


Figure 5.8: The box specific to Pythagoras' Theorem

Based on the semiotic construction in Figure 5.8, different colours are utilized to suggest the difference between the *"Foundation Part"* and *"Non-Foundation Part"* (EDB, 1999, p. ANNEX III) in HKDSE mathematics. The *"Foundation Part"* is white coloured and the *"Non-Foundation Part"* is coloured grey. The spatial distinction between "white" and "grey" is read as a topographical representation of the internal relationship within "Pythagoras' Theorem" representing "the physical spatial relations" (Kress & van Leeuwen, 2006, p. 98) between these two colours.

This differentiation using different colours is inter-textually related with the second instance of Pythagoras' Theorem (EDB, 1999, p. 23) presented in Table 5.2. With reference to Table 5.2, one objective, to "appreciate the dynamic element of mathematics knowledge through studying the story of the first crisis of mathematics" was underlined (___) and was treated as the non-foundation part. This part is indicated in grey in Figure 5.8. Therefore, there is a situation of semiotic adoption (O'Halloran, 2007b) in which two types of non-linguistic resources are used for encoding the meaning of the non-foundation part. One is the use of the emphatic device of "underlines" used in Table 5.2 and the other is the use of colour in Figure 5.8. The meaning of both devices is explained in footnotes. For example, with regard to the meaning of underlines in Table 5.2, the footnote states that "the objectives underlined are considered as **non-foundation** part of the syllabus" (EDB, 1999, p. 23) while for the meaning of the grey colour in Figures 5.6 and 5.8, its footnote highlights the " NON-FOUNDATION PART" (EDB, 1999, p. 23). This is an example of semiotic mixing where colour as one participant, and one nominal group as the other participant, are related to the ellipse of relational process as portrayed in Figure 5.8.

The meaning between the underlined words and the grey colour is bridged and united with encoding from other resources. With respect to Figure 5.8, the *"Foundation part"* and *"non-foundation part*" together form the internal characteristics of Pythagoras' Theorem, indicating a meronymic relationship between Pythagoras' Theorem and its internal characteristics.

The meronymic relationship within Pythagoras' Theorem in Figure 5.8 is identified with the use of the semiotic resources of colour (white and grey), offering a fruitful means of encoding different internal characteristics of Pythagoras' Theorem. This manner of encoding resembles the function of symbolism which encodes meaning to symbols. Linguistic resources have been encoded as different colours.

This compositional relationship indicated through using different colours could be converted into a knowledge structure. The knowledge structure between different mathematical concepts is inspired by Bernstein's (2000) model of the knowledge structure. This adapted model is presented in Figure 5.9.

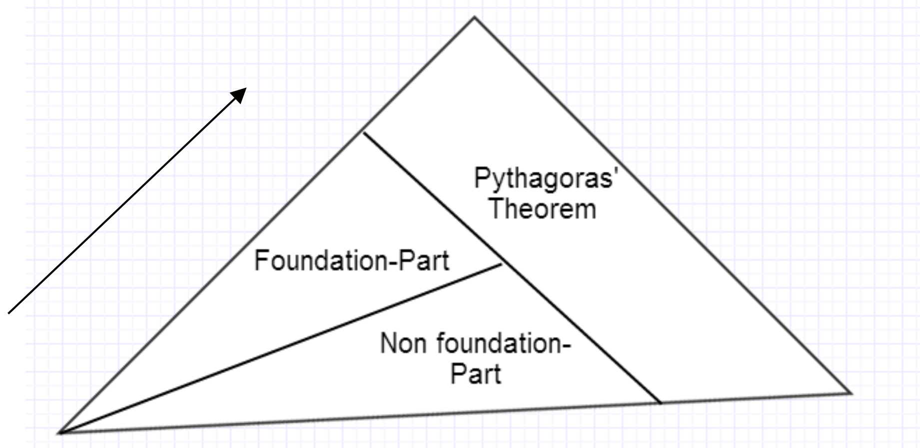


Figure 5.9: Knowledge Structure concerned with Pythagoras' Theorem and its two sub-categories

There are three triangles in Figure 5.9, one for the foundation-part, another for the non-foundation part and the most integrating triangle for Pythagoras' Theorem. Based on Figure 5.9, both the foundation part of Pythagoras' Theorem and the non-foundation part of Pythagoras' Theorem are integrated by Pythagoras' Theorem, generating a hierarchical knowledge structure. Between the foundation part and non-foundation part, their taxonomic relationship is converse, suggesting the horizontal knowledge structure following Bernstein's (1999, 2000) elaboration.

As for the overall meaning-making process in Figure 5.6, the sequential order between different mathematical concepts, which is encoded in the flowchart, implies their knowledge structures as considered by the EDB (1999). Speaking of Pythagoras' Theorem, solid arrows " ↓ " used in the flowchart signal the knowledge progression between "Simple Introduction to Deductive Geometry", "Pythagoras' Theorem" and "Quadrilateral". The relationship between these three mathematical concepts which had not been explicated in Table 5.1 is narrowed to a hierarchical knowledge structure with "Quadrilateral" being the most integrating mathematical concept, followed by "Pythagoras' Theorem" and then by "Simple Introduction to Deductive Geometry". By saying Quadrilateral is the most integrating, I mean Quadrilateral absorbs other levels of mathematical concepts. With respect to the semiotic situation of Pythagoras' Theorem considered in Figure 5.8, the distribution of different colours creates an internal relationship between Figure 5.8 and Table 5.2. The internal characteristics of Pythagoras' Theorem that are linguistically described in Table 5.2, have been encoded as different colours. This way of introducing meaning in a more economic concern for space is an extension of the function of symbolism into the meaning-making process of colour.

5.2.4 Different instances of knowledge representation and forming up the ideational meaning commitment movement in Syllabus (EDB, 1999)

The representations of Pythagoras' Theorem in the Syllabus (EDB, 1999) utilize tabular taxonomy and flowcharts. These two ways of making meaning could be categorized as conceptual representations of knowledge with respect to the understanding of Kress & van Leeuwen (2006, Chapter 3). The functionalities of the tabular taxonomy and flowcharts differ from each other. In tabular taxonomies, "specific concepts are subordinated to more general and abstract concepts" (Kress & van Leeuwen, 2006). As shown in both Table 5. 1 and Table 5.2, the relationship of subordination between Pythagoras' Theorem and other mathematical concepts has been identified. As for flowcharts, they display a "linear representation" of information (Kress & van Leeuwen, 2006, p. 84). Linearity could be understood as information organised in a flowchart having a "sequential progression … with a clear beginning and an end" (Kress & van Leeuwen, 2006, p. 84). Specific to Pythagoras' Theorem, a sequential progression between "Simple Introduction to Deductive Geometry", "Pythagoras' Theorem" and "Quadrilateral" is shown in Figure 5.6 with the help of the default flowchart.

Moving from the meaning-making process in tabular taxonomies and flowcharts to the understanding of how the knowledge structure of Pythagoras' Theorem has been indicated, I am considering Bernstein's (1999, 2000) knowledge structures from the perspective of ideational meaning. Generalisation (Hood, 2008, p. 357), which is one of the devices depicting ideational meaning commitment, has been extended to account for the building of a meronymic relationship between Pythagoras' Theorem and other mathematical concepts.

Both the generating relationship indicated in the tabular taxonomy and the sequential progression indicated in the flowchart, could be treated as the reflection of generalisation indicating a direction of progression, bridging the less integrating mathematical concept with more integrating mathematical concept.

With respect to the correlation between the three instances considered in this section, all the three instances are linguistically cohesive in repeating the same nominal group: "Pythagoras' Theorem". Focusing on the knowledge relationship across these instances, different levels of commitment are carefully organised. The organisation suggests that one of the major functionalities of the Syllabus (EDB, 1999) is to transform the invisible mathematical knowledge and mathematical knowledge structure (Devlin, 1998, preface) into a visible representation In the Syllabus (EDB, 1999), the textual organisation of mathematical visual images such as the tabular taxonomy and the flowchart format help to transform invisible mathematical knowledge and the knowledge structures considered by the Education Bureau of Hong Kong.

Table 5.1 demonstrates the meronymic relationship in the tabular taxonomy with Pythagoras' Theorem being integrated within other mathematical concepts, reflecting a hierarchical knowledge structure. Table 5.2 also demonstrates a meronymic relationship in the tabular taxonomy whereby Pythagoras' Theorem is integrated by its immediate upper level mathematical concept while absorbing other mathematical concepts. There is a relay between Table 5.1 and Table 5.2. The former focuses on the parental structure of Pythagoras' Theorem. For example, "Learning Geometry through a Deductive Approach" integrates "Pythagoras' Theorem". The latter focuses on six subsections that are integrated by "Pythagoras' Theorem". The instance presented in Figure 5.6 summarizes the meronymic relationship in the first two instances and renders a more precise and concise knowledge structure of Pythagoras' Theorem that has been perceived by the EDB (1999). By stating that Figure 5.6 is more precise, I refer to the encoding of sequential knowledge progression using solid arrows " ↓ ". With the assistance of " ↓ ", the relationship between "Simple Introduction to Deductive Geometry", "Pythagoras' Theorem" and "Quadrilateral" that were generally categorised as Vertical Knowledge Structure at a conceptual level in the previous two instances, has now become more precise. A hierarchical knowledge progression between "Simple Introduction to Deductive Geometry", "Pythagoras' Theorem" and "Quadrilateral" could be identified with the assistance of sequential progression portrayed by solid arrows " ↓ ". By stating that Figure 5.6 is more concise, I refer to the use of different colours to condense the lengthy descriptions provided by Table 5.2.

Although I claim a greater precision and conciseness of making meaning has eventuated as shown in Figure 5.6, I do not intend to diminish the importance of Tables 5.1 and 5.2. Actually, the relationship between these three instances is in the form of a relay. This relay comprises a three-staged progression, including: 1) a concise table displaying the mathematical concept as a technical term in the nominal group (EDB, 1999, p. 13); 2) a continuation of the table rendering the internal components within that mathematical concept (EDB, 1999, p. 23); and 3) a summarized flowchart indicating what has been provided in the previous two stages, using an economic form of expression.

A synthesis of the knowledge structure generated by the three independent instances of representation of Pythagoras' Theorem is foregrounded in Figure 5.10, summarizing different knowledge structures outlined in this section.

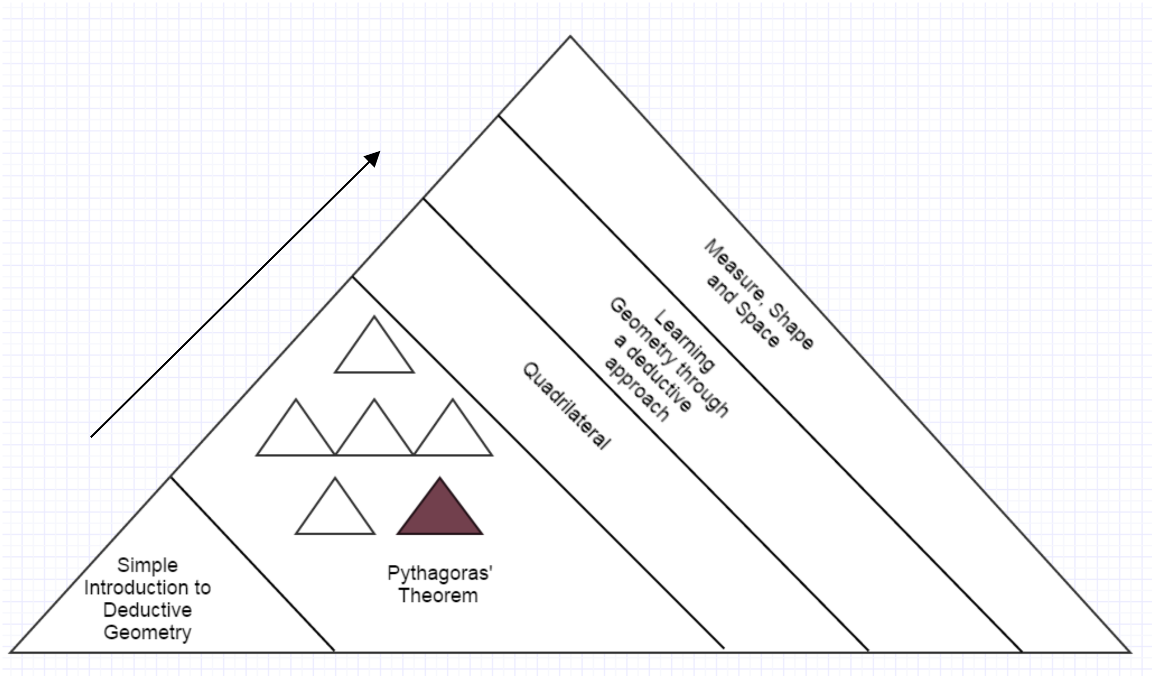


Figure 5.10: A synthesis of the knowledge structure regarding Pythagoras' Theorem in the syllabus (EDB, 1999)

In Figure 5.10, five overlapping triangles together with six proportionally smaller triangles contribute to the knowledge structure regarding how Pythagoras' Theorem has been represented in the Syllabus (EDB, 1999). These five overlapping triangles are displayed based on the function of the tabular taxonomy and flowchart explained in Sections 5.2.2 to 5.2.4. The smallest triangles represent the internal characteristics of Pythagoras' Theorem outlined in Section 5.2.3. A specific grey triangle is also provided, representing one of internal characteristics of Pythagoras' Theorem as a non-foundation part. The arrow indicates the direction of progression moving from the least integrated mathematical concept to the most integrated mathematical concept.

5.3 Ideational commitment in the curriculum guideline: Instance of Pythagoras' Theorem in the curriculum guideline

The curriculum guideline (HKEAA, 2007) explains and expands the Syllabus (EDB, 1999), enhancing the Syllabus with concrete examples. With reference to the curriculum ecology, the curriculum guideline is a relay of the syllabus, explicating the knowledge requirements specified in the syllabus with practical pedagogical applications. Compared to the syllabus, the curriculum guideline is more detailed in terms of both its tight connection with the newly imposed 3-3-4 education policy, and its regulation of other pedagogic activities under this new education system. For example, in the curriculum guideline, the explication is achieved through the provision of the instructional advice for teaching and learning in a classroom setting, the principles for designing textbooks and the regulations for compiling examination papers. These are the reasons why the curriculum guideline has been placed in the policy-making stage, serving as the transition between the Syllabus (EDB, 1999) and other pedagogic activities such as the textbooks (e.g. Wong & Wong, 2007) and the examination papers (e.g. HKEAA, 2012).

Bernstein (1990) interprets the relationship between different pedagogic discourses as a relay. The relay is interpreted as recontextualisation where mathematical knowledge underlined in the Syllabus (EDB, 1999) has been delocated from its original setting which is the Syllabus and thus is relocated into the current target of analysis, the curriculum guideline.

In terms of the research focus of this study, the delocation and relocation of Pythagoras' Theorem has been identified. Pythagoras' Theorem is presented in relation to the question of finding "the properties of scalar product of vectors" (HKEAA, 2007, p. 113). This way of portraying mathematical concepts is tightly associated with the instructional nature of the curriculum guideline. Instructional advice for one mathematical concept, namely "the properties of scalar product of vectors", are provided with Pythagoras' Theorem serving as its prerequisite knowledge.

In the original text (HKEAA, 2007, pp. 113–114), one text box displays how "the properties of scalar product of vectors" could be introduced in a classroom setting. This instance was provided in Chapter Four for a demonstration of one sample of research data. This instance is concerned with an embedded relationship between a linguistic statement – "The example below illustrates how some of the approaches and strategies can be used in the Mathematics classroom" – and a text box that is projected from the linguistic statement. This instance is an independent pedagogic item in which Pythagoras' Theorem is instantiated.

Figure 5.11 introduced this instance, maintaining the original semiotic resources such as bold font and its formal typeface.

The example below illustrates how some of the approaches and strategies can be used in the Mathematics classroom.

Teaching one of the properties of the scalar product of vectors using the direct instruction, the inquiry and the co-construction approaches

Teachers may integrate various teaching approaches and classroom practices to introduce the properties of the scalar product of vectors so that the lessons can be more vivid and pleasurable. In this example, teaching one of the properties of the scalar product of vectors, $\mathbf{|a-b|^2 = |a|^2 + |b|^2 - 2\,(a \bullet b)}$, is used as an illustration.

In previous lessons, the teacher has taught the concepts of magnitudes of vectors and the scalar product of vectors using **direct instruction.** In this lesson, the students are divided into small groups to promote discussion, and the groups are asked to **explore** the geometrical meaning of the property. Here, the **inquiry approach** is adopted, with students having to carry out **investigations** with the newly acquired knowledge related to vectors. During the exploration, the groups may interpret the geometrical meaning differently. Some may consider one of the vectors to be a zero vector and get the above property; but others may relate it to the Pythagoras' Theorem by constructing two perpendicular vectors a and b with the same initial point. Hence, the hypotenuse is $|a-b|$ and $a \bullet b = 0$ and the result is then immediate. If some groups arrive at this conclusion, the teacher should guide them to discover that their interpretation is only valid for special cases. However, the geometrical meaning of this property is related to the cosine formula learned in the Compulsory Part. If some groups can find that the property is the vector version of the cosine formula, they can be invited to explain how they arrived at this geometrical meaning. If none of the groups can arrive at the actual meaning, the teacher may guide them to find it out by giving prompts. Some well-constructed prompts (or scaffolds), such as asking them to draw various types of triangles and find clues to connect |a-b|, a•b, |a| and |b| with triangles drawn, may be provided. The co-construction approach is adopted here.

After understanding the geometrical meaning, the result can be derived by applying the cosine formula learned in the Compulsory Part. The groups are further asked to explore alternative proofs. Here, the inquiry approach is employed. The groups may not think of proving this property with $|x|^2 = x \bullet x$ directly. The teacher may give some hints to guide them. In this case, the teacher and the students are co-constructing knowledge. If the students still cannot prove this property, the teacher can demonstrate the proof on the board using the direct instruction approach. Whatever methods the students use, they are invited to explain their proofs to the class. During the explanation, the teacher and student may raise questions and query the reasoning.

Figure 5.11: Pedagogic Item where Pythagoras' Theorem is instantiated in the curriculum guideline(adapted from HKEAA, 2007, pp. 103–104)

Figure 5.11 is the instructional recommendation from HKEAA (2007) to guide subject teachers in teaching "one of the properties of the scalar product of vectors". In this example, Pythagoras' Theorem has been instantiated as one nominal group at the semiotic level. At the conceptual level, the knowledge structure concerned with Pythagoras' Theorem is instantiated through the taxonomic relationship as well as the logical relationship perceived. A blueprint of analysis underlined above is replicated in the following subsections.

5.3.1 The semiotic situation of this pedagogic item

Starting with the semiotic resources used in this pedagogic item, it is an example of semiotic complex where verbal language co-occurs with mathematical symbolism. The predominant semiotic resource is verbal language, occupying the majority of spaces. A list of mathematical symbols is displayed in Table 5.4.

Table 5.4: Examples of mathematical symbols in Figure 5.11

Mathematical symbols
$\|a-b\|^2 = \|a\|^2 + \|b\|^2 - 2\,(a \bullet b)$
$\|a-b\|$
$a \bullet b = 0$
$\|x\|^2 = x \bullet x$

The semiotic situation of Figure 5.11 is in essence a combination of both verbal language and mathematical symbols. Table 5.4 listed the types of mathematical symbols identified in Figure 5.11. As for the function of mathematical symbols, experiential potentials have been encoded within mathematical symbols allowing them to function as participants, processes and circumstances. From a lexicogrammar perspective, the symbolic expressions are briefer than linguistically rendered versions with the same amount of experiential meaning being encapsulated into the relatively smaller units.

5.3.2 Building up the taxonomic relationships in Figure 5.11

With respect to the commitment of ideational meaning in Figure 5.11, the taxonomic relationship available there needs to be explored with an emphasis on how the technical field regarding "one of the properties of the scalar product of vectors" is established. The building of the technical field is closely associated with the taxonomic relationship between different mathematical concepts in Figure 5.11, among which Pythagoras' Theorem is one of the key components. The purpose in underlining the taxonomic relationship is to unveil how Pythagoras' Theorem is co-related with other mathematical concepts when their conceptual network relationship is established through linguistic and symbolic resources rather than through the diagrammatical visual images that were utilised by the Syllabus (EDB, 1999).

According to Martin & Rose (2014), the taxonomic relationship between different lexical items can be categorised into eight types. A meronymic relationship focuses on the part-whole relationship between two lexical items where A is a part of B. A hypotonic relationship focuses on the class-member relationship between two lexical items where A is a kind of B. These two types of relationship are typically found in technical fields. Regarding the nature of a meronymic hierarchy and a hypotonic hierarchy, Martin and Rose (2014) suggest, "both hierarchies may have many layers" (p. 80), indicating the existence of staged layers within both hierarchies.

Apart from the part-whole relationship expressed through the meronymic hierarchy and the class-member relationship expressed through the hypotonic hierarchy, the taxonomic relationship between different lexical items could also be interpreted as *repetition, synonym, antonym, converse, scales* and *cycles*, each of which describes one type of experiential relations that might be identified between different lexical items. *Repetition* indicates that the same lexical items are repeated across the text based on the textual inference. *Synonym* suggests that two lexical items share a similar experiential meaning such as marriage and wedding. Both *antonym* and *converse* deal with the oppositional experiential meaning between two lexical items. *Antonym* suggests an oppositional experiential meaning in terms of the state and condition such as the comparison between marriage and divorce, and between yin and yang. *Converse* is concerned with the "converse roles" (Martin & Rose, 2014, p. 80) played by different characters, such as the comparison between wife and husband, between teacher and student, and between doctor and patient. The last type of taxonomic relationship between different lexical items is concerned with *series, scales* and *cycles* being its subcategories. *Scales* describe different lexical items that are experientially related with reference to their continuum of states and conditions such as the scale of hot, warm, tepid and cold in scientific discourse. *Cycles* are concerned with lexical items that form a time dimension within the scale, different lexical items appearing in a predesigned order such as the days of the week, and the months of the year.

Based on Martin and Rose (2014), the taxonomic relations system outlined here is diametrically outlined in Figure 5.12.

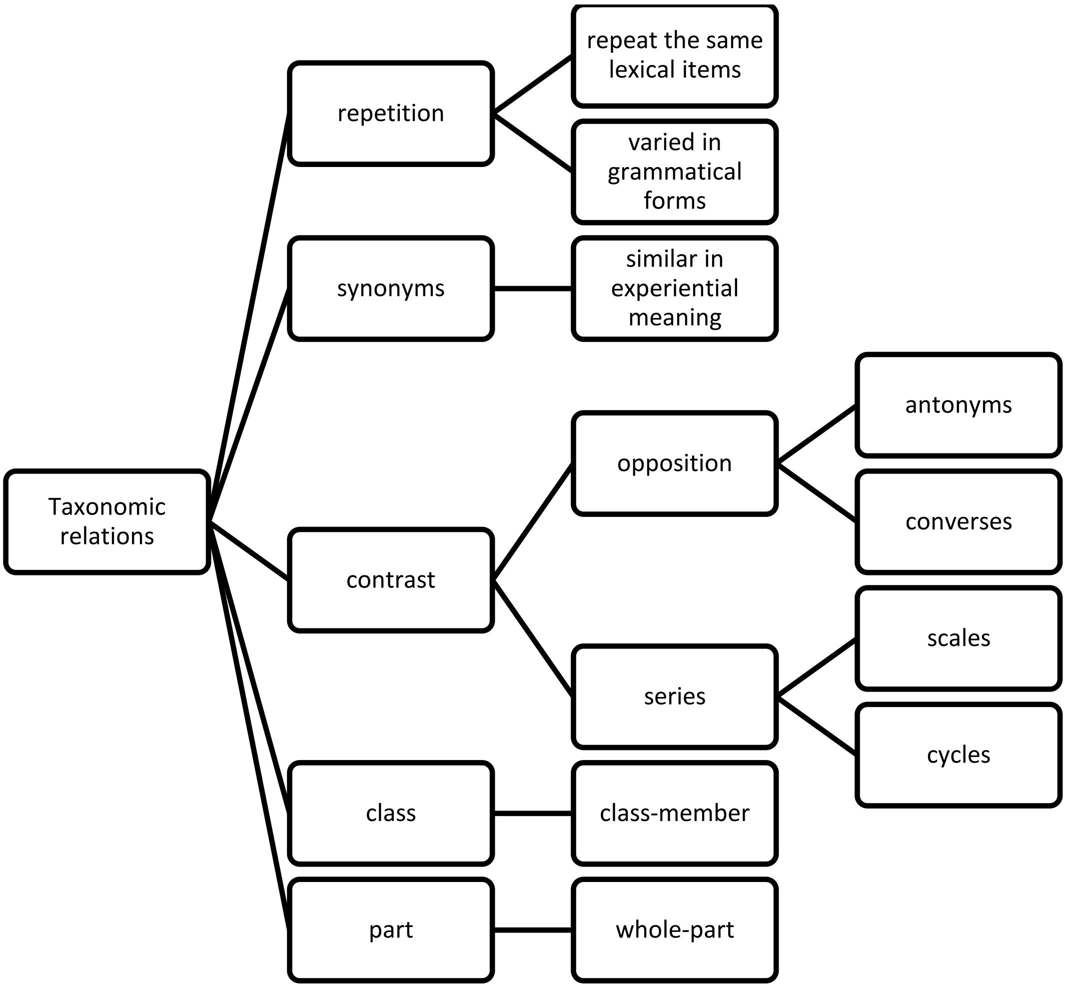


Figure 5.12: Taxonomic relations system (adapted from Martin & Rose, 2014, p. 81)

This system could be applied to underpin the taxonomic relationship system in relation to “one of the properties of the scalar product of vectors” in Figure 5.11. To infer a taxonomic relations system specific to “one of the properties of the scalar product of vectors”, lexical items associated with it are underlined and italicized in Figure 5.13. Each separate line has been labelled with numbers for the convenience of future analysis.

Line #	Wordings in each line
1	Teaching *one of the properties of the scalar product of vectors* using the direct
2	instruction, the inquiry and the co-construction approaches
3	Teachers may integrate various teaching approaches and classroom practices to
4	introduce *the properties of the scalar product of vectors* so that the lessons can
5	be more vivid and pleasurable. In this example, teaching *one of the properties*
6	*of the scalar product of vectors,* $|a-b|^2 = |a|^2 + |b|^2 - 2 (a \bullet b)$, is used as an
7	illustration.
8	In previous lessons, the teacher has taught *the concepts of magnitudes of*
9	*vectors* and *the scalar product of vectors* using direct instruction. In this lesson,
10	the students are divided into small groups to promote discussion, and the
11	groups are asked to explore *the geometrical meaning of the property*. Here, the
12	inquiry approach is adopted, with students having to carry out investigations
13	with *the newly acquired knowledge related to vectors*. During the exploration,
14	the groups may interpret *the geometrical meaning* differently. Some may
15	consider *one of the vectors to be a zero vector* and get *the above property*; but
16	others may relate it to *the Pythagoras' Theorem* by constructing *two*
17	*perpendicular vectors a and b with the same initial point*. Hence, *the*
18	*hypotenuse is* $|a-b|$ and $a \bullet b = 0$ and the result is then immediate. If some groups
19	arrive at this conclusion, the teacher should guide them to discover that their
20	interpretation is only valid for special cases. However, *the geometrical meaning*
21	*of this property* is related to *the cosine formula* learned in the Compulsory Part.
22	If some groups can find that *the property* is *the vector version of the cosine*
23	*formula*, they can be invited to explain how they arrived at *this geometrical*
24	*meaning*. If none of the groups can arrive at the actual meaning, the teacher
25	may guide them to find it out by giving prompts. Some well-constructed
26	prompts (or scaffolds), such as asking them to draw *various types of triangles*
27	and find clues to connect $|a-b|$, $a \bullet b$, $|a|$ *and* $|b|$ *with triangles* drawn, may be
28	provided. The co-construction approach is adopted here.
29	After understanding *the geometrical meaning*, the result can be derived by
30	applying *the cosine formula* learned in the Compulsory Part. The groups are
31	further asked to explore alternative proofs. Here, the inquiry approach is
32	employed. The groups may not think of proving *this property* with $|x|^2 = x \bullet x$
33	directly. The teacher may give some hints to guide them. In this case, the
34	teacher and the students are co-constructing knowledge. If the students still
35	cannot prove *this property*, the teacher can demonstrate the proof on the
36	board using the direct instruction approach. Whatever methods the students
37	use, they are invited to explain their proofs to the class. During the explanation,
38	the teacher and student may raise questions and query the reasoning.

Figure 5.13: Lexical items, associated with "one of the properties of the scalar product of vectors", are underlined and *italicized.*

The inferring of the lexical relationship arises from two perspectives, one for textual inference and the other for ideational inference. One of the lexical relationships, repetition, is achieved through textual inference with the assistance of textual features. This textual inferring suggests that the same lexical items are repeated several times across the text in Figure 5.11. For example, repetition could be identified easily with the repeated lexical items in the following examples: "*one of the properties of the scalar product of vectors"* (Lines 5 & 6), "*the property"* (Line 15), and *"this property"* (Line 32 & Line 35). Lexical strings of repetition are provided from Figure 5.14 to Figure 5.16 with "one of the properties of the scalar product of vectors" being displayed in Figure 5.14, "the geometrical meaning of the property" being displayed in Figure 5.15 and "the cosine formula" being displayed in Figure 5.16. The number in the brackets: "( )" is the label of the line so that the exact location of repetition could be detected. Incidents of taxonomic relationship are labelled in alphabetical order, starting with the capitalized letter A. Each taxonomic relationship is elaborated regarding how the relationship is achieved.

Number of Lines and Taxonomic Relationship	**Lexical Items**	**Line Numbers**
A:	*one of the properties of the scalar product of vectors*	(Line 1)
Taxonomic relationship:	Repetition. The experiential meaning in Incident A is maintained in the following Incident (Incident B) with the existence of repetition of the same lexical items.	
B:	*one of the properties of the scalar product of vectors*	(Line 5 & Line 6)
Taxonomic relationship:	Repetition: The experiential meaning in Incident B is maintained in the following Incident (Incident C) with the existence of repetition of the same lexical items.	
C:	*the above property*	(Line 15)
Taxonomic relationship:	Repetition: The experiential meaning in Incident C is maintained in the following Incident (Incident D) with the existence of repetition of the same lexical item: "property".	
D:	*the property*	(Line 22)
Taxonomic relationship:	Repetition: The experiential meaning in Incident D is maintained in the following Incident (Incident E) with the existence of repetition of the same lexical item: "property".	
E:	*this property*	(Line 32)

Figure 5.14: Lexical string of repetition for "one of the properties of the scalar product of vectors"

Number of Lines and Taxonomic Relationship	Lexical Items	Line Numbers
A:	*the geometrical meaning of the property*	(Line 11)
Taxonomic relationship:	Repetition. The experiential meaning in Incident A is maintained in the following Incident (Incident B) with the existence of repetition of the same lexical items: “the geometrical meaning”.	
B:	*the geometrical meaning*	(Line 14)
Taxonomic relationship:	Repetition. The experiential meaning in Incident B is maintained in the following Incident (Incident C) with the existence of repetition of the same lexical items: “the geometrical meaning”.	
C:	*the geometrical meaning of the property*	(Line 20 & Line 21)
Taxonomic relationship:	Repetition. The experiential meaning in Incident C is maintained in the following Incident (Incident D) with the existence of repetition of the same lexical items: “geometrical meaning”.	
D:	*this geometrical meaning*	(Line 23 & Line 24)
Taxonomic relationship:	Repetition. The experiential meaning in Incident D is maintained in the following Incident (Incident E) with the existence of repetition of the same lexical items: “geometrical meaning”.	
E:	*the geometrical meaning*	(Line 29)

Figure 5.15: Lexical string of repetition for “the geometrical meaning of the property”

Number of Lines and Taxonomic Relationship	Lexical Items	Line Numbers
A:	*the cosine formula*	(Line 21)
Taxonomic relationship:	Repetition. The experiential meaning in Incident A is maintained in the following Incident (Incident B) with the existence of repetition of the same lexical items: “the cosine formula”.	
B:	*the cosine formula*	(Line 30)

Figure 5.16: Lexical string of repetition for “the cosine formula”

Repetition is largely evidenced through textual inference where the same lexical items recurrently appear based on the elaboration from Figures 5.14 to 5.16. The means of inferring repetition through textual resources is obvious based on the semiotic resources provided in the texts.

Textual inference could also be identified in other taxonomic relationships such as inferring how a hypotonic relationship and a meronymic relationship are achieved. For example, with respect to the two nominal groups: "*the properties of the scalar product of vectors"* and *"one of the properties of the scalar product of vectors"*, there is a hypotonic relationship between them with the latter being a kind of the former. The hypotonic relationship between these two nominal groups is signalled through the "Head-Thing" structure in the second nominal group and the repeated lexical items: "*the properties of the scalar product of vectors"* shared by both nominal groups. To be more explicit, in the second nominal group: "*one of the properties of the scalar product of vectors*", "one of" is the Head of this nominal group, while "*the properties of the scalar product of vectors"* is the Thing. Within a nominal group, "Head" occupies only parts of the "Thing" or represents only kinds of the "Thing" (Halliday & Matthiessen, 2004). In this example, what has been occupied by the "Head" is a kind of element within the "Thing", addressing the hypotonic relationship between "Head" and "Thing". Since the "Thing": "*the properties of the scalar product of vectors*" is the whole wording of the first nominal group, it is justifiable to announce that the relationship between "*one of the properties of the scalar product of vectors*" and "*the properties of the scalar product of vectors*" is achieved through a hypotonic relationship with the former being a kind of the latter.

Similarly, a two-staged meronymic relationship could be identified between *"the properties of the scalar product of vectors"* and *"the scalar product of vectors",* with the former addressing elements within the latter. In this type of comparison, "the properties" as the "Head" in the first nominal group accounts for parts of the elements within the "Thing": "the scalar product of vectors". In a similar manner, a two-staged meronymic relationship could be identified between "*the geometrical meaning of the property"* and *"the property"* with the former being a part of the latter and dealing with only the "geometrical meaning" of the latter.

Figure 5.17 demonstrates the lexical strings concerned with the hypotonic relationship and meronymic relationship regarding "*one of the properties of the scalar product of vectors*". This figure also incorporates the relationship of repetition articulated in Figure 5.7, forming a whole picture regarding the introduction of *"one of the properties of the scalar product of vectors"* in Figure 5.3.1. Incidents of taxonomic relationship are labelled in alphabetical order, starting with the capitalized letter A. Each taxonomic relationship is elaborated regarding how the relationship is achieved.

Number of Lines and Taxonomic Relationship	Lexical Items	Line Numbers
A:	*one of the properties of the scalar product of vectors*	(Line 1)
Taxonomic relationship:	hypotonic relationship: Incident A and Incident B are organized as hypotonic relationship with Incident A being a kind of Incident B.	
B:	*the properties of the scalar product of vectors*	(Line 4)
Taxonomic relationship:	hypotonic relationship: Incident B and Incident C are organized as hypotonic relationship with Incident C being a kind of Incident B.	
C:	*one of the properties of the scalar product of vectors*	(Lines 5 & 6)
Taxonomic relationship:	Repetition: The experiential meaning in Incident C is maintained in the following Incident (Incident D) with the existence of repetition of the same lexical item (property)	
D:	*the above property*	(Line 15)
Taxonomic relationship:	Meronymic relationship: Incident D and Incident E are organized as meronymic relationship with Incident E being a part of Incident D.	
E:	"*the geometrical meaning of the property"*	(Lines 20 & 21)
Taxonomic relationship:	Meronymic relationship: Incident E and Incident F are organized as meronymic relationship with Incident E being a part of Incident F.	
F:	*the property*	(Line 22)
Taxonomic relationship:	Repetition: The experiential meaning in Incident F is maintained in the following Incident (Incident G) with the existence of repetition of the same lexical item (property)	
G:	*this property*	(Line 32)

Figure 5.17: Lexical string of repetition, hyponymy and meronomy for "one of the properties of the scalar product of vectors"

Figure 5.17 indicates a lexical string concerning *"one of the properties of the scalar product of vectors"* with respect to the three lexical relationships: repetition, hyponymy and meronomy identified in Figure 5.11.

Figure 5.17 could be merged with Figure 5.15 to suggest the lexical relationship between three nominal groups: *"one of the properties of the scalar product of vectors"*, *"the properties of the scalar product of vectors"* and *"the geometrical meaning of the property"* based on the textual resources at hand. This merged figure is presented in Figure 5.18 with a tiered structured embedded to signal the compositional relationship (hypnotic relationship and meronymic relationship). Incidents of taxonomic relationship are labelled in alphabetical order, starting with the capitalized letter A. Each taxonomic relationship is elaborated regarding how the relationship is achieved.

Number of Lines and Taxonomic Relationship	**Tier One**	**Tier Two**	**Tier Three**	**Line Number**
A:		*one of the properties of the scalar product of vectors*		Line 1
Taxonomic relationship:		hypotonic relationship: Incident A and Incident B are organized as hypotonic relationship with Incident A being a kind of Incident B.		
B:	the properties of the scalar product of vectors			Line 4
Taxonomic relationship:	hypotonic relationship: Incident B stands at overarching tier (Tier One). Incident B and Incident C are organized as hypotonic relationship with Incident C being a kind of Incident B.			
C:		one of the properties of the scalar product of vectors		Lines 5 & 6
Taxonomic relationship:		Meronymic relationship: Incident C is the second Tier. Incident C and Incident D are organized as meronymic relationship with Incident D being a kind of Incident C.		
D:			The geometrical meaning of the property	Line 11
Taxonomic relationship:			Repetition	
E:			The geometrical meaning	Line 14

Taxonomic relationship:		Repetition: The experiential meaning in Incident E is maintained in the following Incident (Incident G) with the existence of repetition of the same lexical item (The geometrical meaning). Both of these incidents are at the third Tier. Based on the inferring of Repetition between Incident E and Incident G. the taxonomic relationship between Incident E and Incident F could be deducted. Although based on the provided textual resources, these two nominal groups either do not share the same lexical items, or form a Head-Thing structure, their relationship is still a Meronymic relationship. Incident F is the second Tier. Incident E is a part of Incident F, suggesting a meronymic relationship.	
F:	the above property		Line 15
Taxonomic relationship:	Meronymic relationship: Incident F is the second Tier. Incident G is a part of Incident F, organised with Incident F as meronymic relationship.		
G:		The geometrical meaning of the property	Lines 20 & 21
Taxonomic relationship:		Meronymic relationship: Incident G is the third Tier. Incident G and Incident H are organized as meronymic relationship with Incident G being a part of Incident H.	
H:	The property		Line 22
Taxonomic relationship:	Meronymic relationship: Incident H is the second Tier. Incident H and Incident I are organized as meronymic relationship with Incident I being a part of Incident H.		
I:		This geometrical meaning	

Taxonomic relationship:		Repetition: The experiential meaning in Incident I is maintained in the following Incident (Incident J) with the existence of repetition of the same lexical item (geometrical meaning)	
J:		The geometrical meaning	Line 29
Taxonomic relationship:		Meronymic relationship: Incident K is the second Tier. Incident J and Incident K are organized as meronymic relationship with Incident J being a part of Incident K.	
K:	This property		Line 32

Figure 5.18: Taxonomic relationship identified between different lexical items

As can be inferred from Figure 5.18, apart from the repetition relationships, a two-staged embedment can be identified based on the three nominal groups. These nominal groups are discerned and associated through repeating the lexical items to create a lexical relationship of repetition, or through establishing the "Head" to create the lexical relationship of either a hypnotic relationship or a meronymic relationship. Therefore, textual resources are suggestive of the experiential relationships between these three nominal groups outlined in Figure 5.18.

The experiential relationships in terms of how different lexical items are associated could be inferred based on the textual resources manifested as linguistic evidence. However, in incidents where the textual relationships between different lexical items are not presented such as the absence of repetition, or they do not share the same lexical items, their experiential relationships need to be inferred from the contextual features.

The inferring of a lexical relationship in the curriculum guideline is different from the ways applied in the Syllabus (EDB, 1999). In the Syllabus, the textual layout, such as the use of tables and diagrams, suggests that lexical relationships are fully supported by textual evidence. Here, in this example, contextual features need to be considered when the lexical relationship has been encoded into activity sequences that are a "series of events that are expected by a field" (Martin & Rose, 2014, p. 101). The inferring of a lexical relationship is based on the assumption that readers are not familiar with the field of knowledge but are literate in terms of the language. In this study, the language is English. The taxonomic relationships between different lexical items are judged by the contextual features available in the pedagogical discourse. In this example, the taxonomic relationship between some nominal groups is judged based on the contextual features in the curriculum guideline (HKEAA, 2007).

The activity sequences in Figure 5.11 are achieved through different mechanisms. For example, the internal relationship between these circumstantial features is suggestive of the activity sequences. A sequential relationship can be identified in the comparison between "*in previous lessons*" at Line 8 and "*in this lesson*" at Line 9. Logically speaking, field-oriented features presented "in previous lessons" occur earlier than those presented "in this lesson", formulating "cohesive successive conjunctions" (Martin & Rose, 2014, p. 126). Therefore, in terms of the sequence of occurrences, "*the concepts of magnitudes of vectors*" (Lines 8 & 9) and "*the scalar product of vectors*" (Line 9) precede "*the geometrical meaning of the property*" (Line 11). "*The concepts of magnitudes of vectors*" (Lines 8 & 9) are the linguistically rendered version of "$|a|^2 + |b|^2$", while "*the scalar product of vectors*" is meronomically associated with "*the properties of the scalar product of vectors*" with the latter being a part of the former. According to the curriculum guideline (HKEAA, 2007), these two fragments of mathematical concepts are supposed to be introduced in previous lessons before advancing to "*the geometrical meaning of the property*". The sequential relationship indicated through the relationship between circumstantial features such as, "*in previous lessons*" and "*in this lesson*", is suggestive of the experiential relationship between these lexical items with "*the concepts of magnitudes of vectors*" (Lines 8 & 9) and "*the scalar product of vectors*" (Line 9) being meronomically associated with "*the geometrical meaning of the property*" (Line 11).

Two mathematical concepts are part of "*the geometrical meaning of the property*" (Line 11). The first is "*one of the vectors to be a zero vector*" (Line 15) and the other is "*the Pythagoras' Theorem*" (Line 16). Their relationship is constructed through dividing the actors: "*students*" (Line 12) into two separate groups: "*some*" (Line 14) and "*others*" (Line 16), each of the group carrying parts of the totality.

With the assistance of logical meaning and contextual features, another lexical string associated with "the geometrical meaning of the property" (Line 11) can be presented in Figure 5.19. Incidents of taxonomic relationship are labelled in alphabetical order, starting with the capitalized letter A. Each taxonomic relationship is elaborated regarding how the relationship is achieved.

Number of Lines and Taxonomic Relationship	**Lexical Items**	**Line Numbers**
A:	*"the geometrical meaning of the property"*	(Line 11)
Taxonomic Relationship	Meronymic Relationship: Both Incident B and Incident C are parts of Incident A, formulating a meronymic relationship. This relationship is identified not through lexical repetition of through Head-Thing structure, but through the circumstantial features associated with these three nominal groups.	
B:	*one of the vectors to be a zero vector*	(Line 15)
Taxonomic Relationship	Converse relationship: With reference to Incident B and Incident C, each of them represents a part of Incident A. Regarding the relationship between them, they form up a converse relationship resembling with the "yin and yang" analogy. The inferring of this taxonomic relationship could not be directly inferred based on the textual resources. Rather, the inferring is achieved through the contextual features as well.	
C:	*The Pythagoras' Theorem*	(Line 16)

Figure 5.19: Lexical string of "the geometrical meaning of the property"

Figure 5.19 could be merged with Figure 5.18 to generate a lexical string in which Pythagoras' Theorem is concerned. The merged figure to be presented in Figure 5.20 suggests how the taxonomic relationships between different lexical items are established through the merge of both a logical perspective and a contextual perspective. Figure 5.20 offers a comprehensive picture in relation to the lexical relationship concerned with "the geometrical meaning of the property". Incidents of taxonomic relationship are labelled in alphabetical order, starting with the capitalized letter A. Each taxonomic relationship is elaborated in terms of how the relationship is achieved.

Number of Lines and Taxonomic Relationship	Tier One	Tier Two	Tier Three	Tier Four	Line Number
A:		*one of the properties of the scalar product of vectors*			Line 1
Taxonomic Relationship		hypotonic relationship: Incident A is a kind of Incident B, forming up a hypotonic relationship			
B:	the properties of the scalar product of vectors				Line 4
Taxonomic Relationship	hypotonic relationship: Incident C is a kind of Incident B, forming up a hypotonic relationship				
C:		one of the properties of the scalar product of vectors			Lines 5 & 6
Taxonomic Relationship		Meronymic relationship: Incident D is a part of incident C, forming up a meronymic relationship			
D:			The geometrical meaning of the property		Line 11
Taxonomic Relationship			Repetition: The taxonomic relationship for Incident D and Incident E is repetition because the same lexical items (the geometrical meaning) is repeated in both.		
E:			The geometrical meaning		Line 14
Taxonomic Relationship			Meronymic Relationship: Both Incident F and Incident G are parts of Incident E, forming up a meronymic relationship.		
F:				One of the vectors to be a zero vector	Line 15

Number of Lines and Taxonomic Relationship	**Tier One**	**Tier Two**	**Tier Three**	**Tier Four**	**Line Number**
Taxonomic Relationship				Converse Relationship: The taxonomic relationship between Incident F and Incident G is a converse relationship because they together forming up the components within Incident E.	
G:				Pythagoras' Theorem	Line 16
Taxonomic Relationship		Repetition: Incident H is bridged with Incident C, forming up a taxonomic relationship of repetition because they commit the same experiential meaning through lexical cohesion.			
H:		the above property			Line 15
Taxonomic Relationship		Meronymic relationship: Incident I is a part of Incident H, forming up a meronymic relationship.			
I:			The geometrical meaning of the property		Lines 20 & 21
Taxonomic Relationship			Meronymic relationship: Incident I is a part of Incident J, forming up a meronymic relationship.		
J:		The property			Line 22
Taxonomic Relationship		Meronymic relationship: Incident K is a part of Incident J, forming up a meronymic relationship.			
K:			This geometrical meaning		
Taxonomic Relationship			Repetition: The relationship between Incident K and Incident L is repetition because the same lexical items are repeated.		
L:			The geometrical		Line 29

Number of Lines and Taxonomic Relationship	Tier One	Tier Two	Tier Three	Tier Four	Line Number
			meaning		
Taxonomic Relationship			Meronymic relationship: Incident I is a part of Incident M, forming up a meronymic relationship.		
M:		This property			Line 32

Figure 5.20: A merged figure of comprehensive lexical strings regarding "the geometrical meaning of the property"

5.3.3 Updated taxonomic relationships and the incorporation of mathematical symbolism.

The taxonomic relationships outlined in Section 5.3.2 focused on how lexical items regarding "the geometrical meaning of the property" are connected as lexical strings enabled by different types of taxonomic relationships. In this section, mathematical symbolism is considered in line with the lexical relationship identified in Figure 5.11. Figure 5.21 is the extract regarding the occurrence of symbolic equations, preserving the original numbering in Figure 5.11.

5	be more vivid and pleasurable. In this example, teaching *one of the properties*
6	*of the scalar product of vectors, $|a-b|^2 = |a|^2 + |b|^2 - 2\ (a \bullet b)$,* is used as an
7	illustration.

Figure 5.21: The extract regarding the occurrence of symbolic equations

The relationship between the clause "teaching *one of the properties of the scalar product of vectors*" and symbolic equation "$|a-b|^2 = |a|^2 + |b|^2 - 2\ (a \bullet b)$", is organised as a paratactic clause structure, in the form of elaboration. Based on this example, the type of elaboration is "exemplification" because the symbolic equation "develops the thesis of the primary clause by becoming more specific about it, often citing an actual example" (Halliday & Matthiessen, 2004, p. 398). In this example, the symbolic equation specifies "*one of the properties of the scalar product of vectors"* by providing a specific example. Based on this example, the notion of elaboration could be extended to cover symbolic equations in mathematical discourse.

16	others may relate it to *the Pythagoras' Theorem* by constructing *two*
17	*perpendicular vectors a and b with the same initial point*. Hence, *the*
18	*hypotenuse is |a-b|* and *a•b = 0* and the result is then immediate. If some groups

Figure 5.22: The second extract related to the mathematical symbolism in Figure 5.11

The other extract related to mathematical symbolism is presented in Figure 5.22. At the sentence level, the "causal-conditional" relationship (Halliday & Matthiessen, 2004, p. 543) between two sentences could be identified with the first sentence standing as the condition of the second sentence, connected by the conjunctive adjunct "hence". The "causal-conditional" relationship between these two sentences is displayed in Figure 5.23

Sentence One	Conjunctive Adjunct	Sentence Two
others may relate it to *the Pythagoras' Theorem* by constructing *two perpendicular vectors a and b with the same initial point*.	Hence	*the hypotenuse is |a-b|* and *a•b = 0* and the result is then immediate

Figure 5.23: The "causal-conditional" relationship between these two sentences

Within a casual-conditional relationship, the conjunction "hence" indicates that "one event obligates another to happen, as cause and effect" (Martin & Rose, 2014, p. 128). In this example, the event preceding "hence" is the "cause" while the events following "hence" are the "effect". There is the obligatory relation (Martin & Rose, 2014, p. 128) between the events in both sentences with the events in the "effect" being dependent on those in the "cause". The event is realized by "the lexical process in a verbal group" (Martin & Rose, 2014, p. 97).

Sentence One		**Conjunctive Adjunct**	**Sentence Two**	
others may relate it to *the Pythagoras' Theorem* by constructing *two perpendicular vectors a and b with the same initial point*.			*the hypotenuse is* $\lvert a\text{-}b \rvert$ and *a•b = 0* and the result is then immediate	
Event	verbal group		Event	verbal group
relate	relate it to the Pythagoras' Theorem	Hence	is	is $\lvert a\text{-}b \rvert$
constructing	constructing two perpendicular vectors a and b with the same initial point"		is	is then immediate
			= (verbalized as equates)	=0

Figure 5.24: Event identification in these sentences

Normally, the identification of events is concerned with "the lexical process in a verbal group" (Martin & Rose, 2014, p. 97). In the features identified in Figure 5.24, the scope of events has been extended to include a symbolic process, such as the relational process indicated by "=" symbol. Relying on the causal-conditional relationship, events in the "effect" could occur only when the events in the "cause" have been accomplished. Therefore, obviously, there are temporal sequences between the events in the "cause" and those in the "effect" with those in the former preceding those in the latter because "each succeeding effect is implied by the preceding cause" (Martin & Rose, 2014, p. 102). In science fields, this "cause and effect" relationship is the "unmarked relation between events in a sequence" (Martin & Rose, 2014, p. 102), and this relationship is known as the "implication sequences" (Martin & Rose, 2014, p. 102).

In this example, the event *"relate it to the Pythagoras' Theorem by constructing two perpendicular vectors a and b with the same initial point"* precedes *"the hypotenuse is |a-b| and a•b = 0 and the result is then immediate"*, constructing a cause-effect relationship. Within this event, an embedded relationship can be identified with the prepositional phrase "*by constructing two perpendicular vectors a and b with the same initial point*" being embedded within "*relate it to the Pythagoras' Theorem*". Another event could also be identified in this prepositional phrase. This embedded prepositional phrase is anchored to the major phrase through enhancing the major phrase by addressing the "the special semantic feature of 'narrowing'" (Halliday & Matthiessen, 2004, p. 496). In this case, the semantic feature of Pythagoras' Theorem has been narrowed as a specific case labelled by the two English alphabetic letters "a" and "b". This is the specific case of the usage of Pythagoras' Theorem. *"Perpendicular vectors with the same initial point"* should be one of the properties of *"Pythagoras' Theorem"*. However, regarding the specific vectors "a & b", Pythagoras' Theorem is the premise to account for "*two perpendicular vectors a and b with the same initial point*".

Drawing from the previous elaborations, Figures 5.21 and 5.22 have been converted into Figures 5.25 and 5.26 respectively to suggest the tiered structure regarding mathematical symbolism identified in this pedagogic item. Incidents of taxonomic relationship are labelled in alphabetical order, starting with the capitalized letter A. Each taxonomic relationship is elaborated with regard to how the relationship is achieved.

Number of Lines and Taxonomic Relationship	**Semiotic Items**	**Line Numbers**
A:	*one of the properties of the scalar product of vectors*	(Lines 5 and 6)
Paratactic relationship	Elaboration: Incident B as exemplification is provided to exemplify a specific example of Incident A	
B:	$|a\text{-}b|^2 = |a|^2 + |b|^2 – 2\ (a\bullet b)$	(Line 6)

Figure 5.25: Paratactic relationship between "one of the properties of the scalar product of vectors" and "$|a\text{-}b|^2 = |a|^2 + |b|^2 – 2\ (a\bullet b)$"

Number of Lines and Taxonomic Relationship	Semiotic Items	Line Numbers
A:	*The Pythagoras' Theorem*	(Line 16)
Enhancement	The semantics in Incident A has been narrowed to account for specific incidents presented in Incident B	
B:	*Two perpendicular vectors a and b with the same initial point*	(Line 17)
Casual-conditional relationship	Causality: *The combination of Incident A and Incident B is the premise for Incident C, Incident D and Incident E*	
C:	*hypotenuse is |a-b|*	(Line 18)
D:	*a•b = 0*	(Line 18)
E:	the result	(Line 18)

Figure 5.26: Casual-conditional relationship identified in Figure 5.11

5.3.4 Bridging semiotic resources with knowledge structure

So far, the taxonomic relationship regarding *"one of the properties of the scalar product of vectors"* has been investigated with parameters taken from both the lexical approach and the clausal approach. Within the lexical approach, both textual inferring and contextual inferring are useful in identifying the taxonomic relationship. Within the clausal approach, the paratactic structure and hypotactic structure are both suggestive of the taxonomic relationship. These SFL-oriented parameters are unified for the same purpose, namely to underline the taxonomic relationship associated with the central theme *"one of the properties of the scalar product of vectors"* on which this pedagogic item focuses. A tiered taxonomic relationship concerning this mathematical concept could be identified. Regarding *"Pythagoras' Theorem"*, it is nested within *"one of the properties of the scalar product of vectors"* as its fourth layer decedent mathematical concept as revealed based on the delicate structure outlined in Figure 5.27.

These tiered structures (Figure 5.27) generated from the analysis are informative of the sociological approach of knowledge representation.

Taxonomic relationships revealed at the lexical level and the clausal level could be converted to suggest the knowledge structure at a conceptual level. Drawn from Bernstein's (2000) model, these structures are suggestive of the vertical discourse structure highlighting that different mathematical concepts are co-related. The taxonomic relationship for lexical relationship is revisited in Figure 5.27. This model has been extended to include Bernstein's (2000) vertical discourse and two subcategories within vertical discourse, namely horizontal knowledge structure and hierarchical knowledge structure.

Knowledge structures	**Taxonomic Relationship**		
	Tier 1	**Tier 2**	**Tier 3**
horizontal knowledge structure	repetition	repeat the same lexical items	
		varied in grammatical forms	
	synonym	similar in experiential meaning	
	contrast	opposition	antonyms
			converses
		series	scales
			cycles
hierarchical knowledge structure	Hyponym	class-member	
	Meronym	Part-whole	

Figure 5.27: Taxonomic relationship included within knowledge structure

At the lexical level, repetition, synonym and contrast together with their embedded subcategories are suggestive of the horizontal knowledge structure, while hyponym and meronym are suggestive of the hierarchical knowledge structure.

Inherited from the bridging between taxonomic relationships within the knowledge structure, in this example, a paratactic relationship is informative of the horizontal knowledge structure while a hypotactic relationship is informative of the hierarchical knowledge structure. At the causal-conditional relationship level, cause as the premise of the effect logically precedes effect.

Figure 5.28 displays the knowledge structure converted from the taxonomic relationship, paratactic relationship, hypotactic relationship and causal-conditional relationship in Figure 5.11. This knowledge structure follows the model provided by Bernstein (2000) with *"the properties of the scalar product of vectors"* being the upmost layer of the conceptual network, encompassing the rest of mathematical concepts appearing in this pedagogic item.

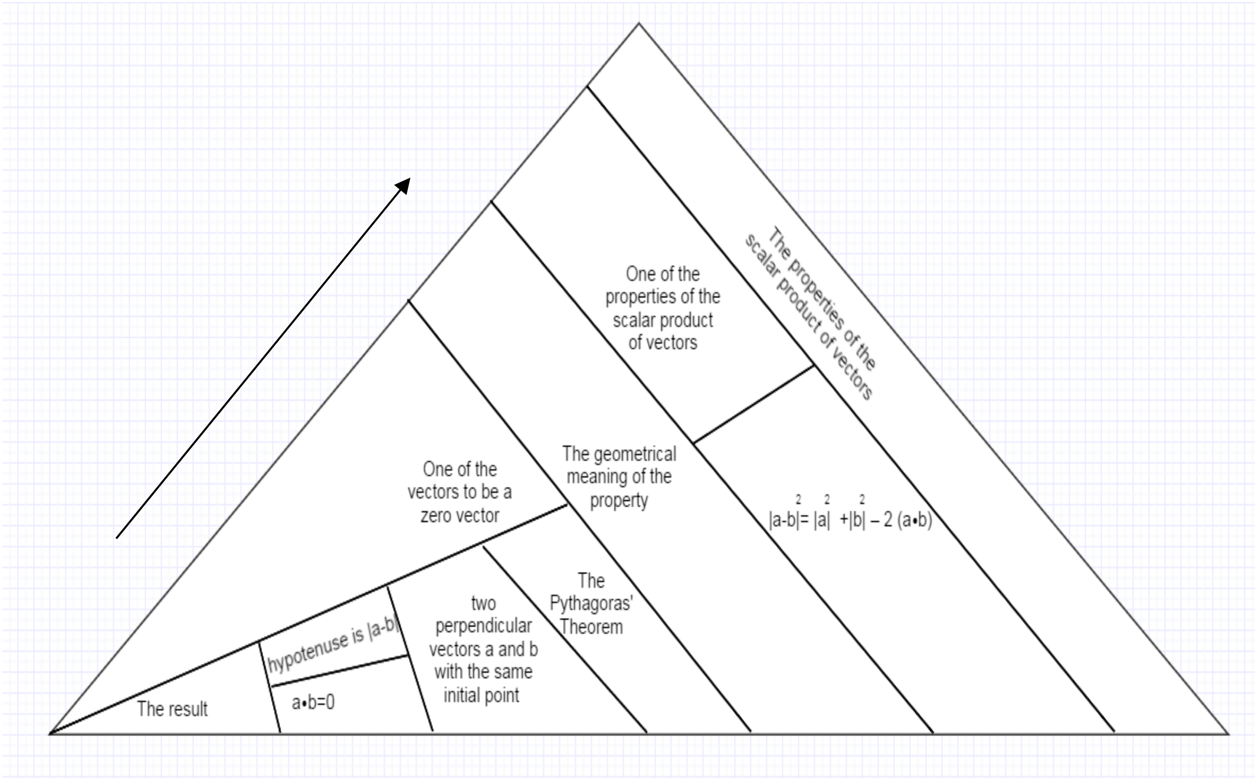


Figure 5.28: Knowledge structure converted from the relationship at lexical level and clausal level

Based on the spatial construction, the pairs of the knowledge relationship regarding *"one of the properties of the scalar product of vectors"* displayed in Figure 5.28 could be interpreted as a combination of both horizontal and hierarchical knowledge structures. With reference to *"Pythagoras' Theorem"*, it is co-related with *"one of the vectors to be a zero vector"* in the form of the horizontal knowledge structure. This knowledge structure at conceptual level is projected from the converse relationship at lexical level. It also subsumes a specific incident of two perpendicular vectors "a" and "b", indicating that the semantics in Pythagoras' Theorem have been narrowed to account for that specific incident. Pythagoras' Theorem in this pedagogic item has also been aborted comprehensively by *"the geometrical meaning of the property"*. This comprehensive absorption at the conceptual level is transformed through the meronymic relationship between them at the lexical level.

5.3.5 Regarding the ideational commitment of Pythagoras' Theorem in this instance

The semiotic resource adopted in presenting Pythagoras' Theorem in the curriculum guideline (HKEAA, 2007) is through a nominal group only. However, the commitment of ideational meaning in terms of how the knowledge structure of Pythagoras' Theorem is established is complicated. This complication lies in the manner of introducing the taxonomic relationship associated with Pythagoras' Theorem. Taxonomic relationships such as repetition, converse, hypotonic relationship and meronymic relationship are coincident with clausal relationships such as a paratactic relationship and a causal-conditional relationship. The combined efforts arising from a range of different relationships inform how different mathematical concepts are co-related.

The identification of the lexical relationship is achieved in two ways. The first is through textual resources evidenced in the repeated lexical items and the specification of the Head in the nominal groups. The second is through logical resources where no salient evidence of textual resemblance could be found. The identification of the paratactic relationship and causal-conditional relationship is achieved through the logical relationship between different events associated with different processes.

The taxonomic relationships identified at the semiotic level are informative of the sociological approach of knowledge construction. A network of conceptual knowledge structure can be sub-categorized into two perspectives: horizontal knowledge construction and hierarchical knowledge construction. With reference to Pythagoras' Theorem, its semiotic construction in the curriculum guideline (HKEAA, 2007) commits both horizontal knowledge construction and hierarchical knowledge construction. The delicate structure regarding how different mathematical concepts are associated within this pedagogic item has been provided in Figure 5.28.

Based on the analysis of the curriculum guideline (HKEAA, 2007), one crucial issue that needs to be brought up here is that we have a tendency in pre-designing and presuming the experiential relationship between lexical items that are textually remote (no incidents of repetition). The experiential relationship needs to be worked out based on the resources at hand rather than jumping to a random assumption in pre-defining the lexical relationship and foreshadowing to the knowledge relationship between different lexical items.

5.4 Introduction to the pedagogic item

In this section, the pedagogic item of assessment is concerned. As has been explicated by the suggested answer to the examination paper prepared also by HKEAA (2012), the required knowledge in this pedagogic item is Pythagoras' Theorem. That is to say, as designed by HKEAA (2012), the knowledge of Pythagoras' Theorem is the pre-requisite knowledge built on which the solution to this assessment could be derived.

During the examination, although Pythagoras' Theorem is the prompt to the assessment, it was never provided to students. Students needed to find the clue leading to Pythagoras' Theorem based on their own conceptual knowledge and the available resources in the assessment. This requires them to delocate the knowledge of Pythagoras' Theorem from the pedagogic texts that they are already familiar with (such as the pedagogic item in textbooks) and relocate the knowledge into the current pedagogic text (the pedagogic item presented as the assessment task) assisted by the mathematical algorithms and laws. This recontextualisation is invisible, however efforts could be devoted to capturing this recontextualisation through working out how knowledge has been delocated from one pedagogic text and relocated into other pedagogic texts. In a social semiotic approach, this process is termed re-instantiation with the ideational meaning commitment being the analytical model in underlining this invisible process.

The pedagogic item in Figure 5.29 is designed for assessment. A textual segmentation allows the pedagogic item to be divided into three independent components, namely: a statement, a geometric image and a scale of multiple choice. To some extent, each of the three independent components is an instance of the instantiation of Pythagoras' Theorem. Pythagoras' Theorem is intersected with other mathematical knowledge at the sociological perspective. The semiotic resources used to explore Pythagoras' Theorem are also in constant interaction with other semiotic resources. The exploration of this pedagogic item helps us to understand one of the most puzzling units of analysis in education – the assessment. The designing of the assessment task is fixed. Different components within the assessment intersect with others; meanwhile each component has its own emphasis and functions.

18. In the figure, $AB = 4$ cm , $BC = CD = DE = 8$ cm and $FG = 9$ cm . Find the perimeter of ΔAEH .

A. 60 cm

B. 74 cm

C. 150 cm

D. 164 cm

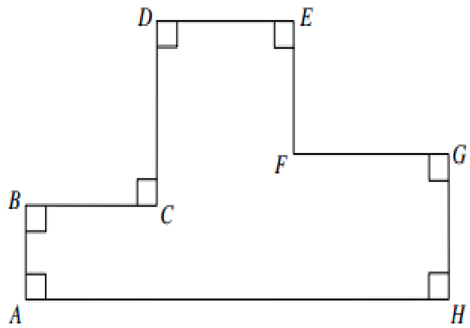


Figure 5.29: The pedagogic item of assessment

Figure 5.29 is a pedagogic item where Pythagoras' Theorem is represented. However, the representation of Pythagoras' Theorem here is different from other data analysed in this study. In other selected examples, such as in the Syllabus (EDB, 1999) and HKEAA (2007), Pythagoras' Theorem as a mathematical concept has been textually rendered as a technical term. Here in this example, the textually rendered version of this mathematical concept has been concealed without a clear indication of the technical term.

The analysis in this section is slightly different from the analysis concerned with the Syllabus (EDB, 1999) and the curriculum guideline (HKEAA, 2007) according to the clear textual cuts between different components. Within each component, the blueprint outlined in Chapter Four will also have been imposed.

5.4.1 The ideational commitment in the statement as the first component

The analysis starts with Figure 5.30, which is concerned with the statement

In the figure, AB = 4 cm, BC = DE = CD = 8 cm, and FG = 9 cm. Find the perimeter of ΔAEH

Figure 5.30: The statement in the pedagogic item

Drawing from the blueprint underlined in Chapter Four, the priority of analysis is to consider its semiotic situation. In Figure 5.30, the semiotic situation is an example of a semiotic complex where linguistic resources coexist with mathematical symbolism.

A linguistically rendered version of Figure 5.30 is presented in Figure 5.31 with mathematical symbols: "=" being verbalised as "equals" and mathematical symbol "Δ" being converted into the linguistic nominal group, "Triangle".

In the figure, AB equals 4 cm, BC equals DE equals CD equals 8 cm, and FG equals 9 cm. Find the perimeter of Triangle AEH

Figure 5.31: The linguistically rendered version of the statement in the pedagogic item

The comparison between the original version and the linguistically rendered version is suggestive of the existence of semiotic adoption (O'Halloran, 2007a and p. 64 in this Phd thesis) where mathematical symbolism displays the experiential content after the encoding of experiential potential from verbal language, for example symbol "=" denotes "process". The semiotic adoption identified between mathematical symbolisms and linguistic resources highlights the fact that mathematical symbolism simplifies the expressions through encoding the experiential process: “equate” and experiential participant: “Triangle” into symbolic forms of “=” and “Δ” respectively. In order to consider how the ideational meaning is committed, choices for process, participants and circumstances are foregrounded in Figure 5.32, because “the construction of experience takes the form of choices for process, participants and circumstance” (O’Halloran, 2005, p. 75).

In the figure, AB = 4cm, BC = DE = CD = 8cm, and FG = 9 cm.

circumstance:	Participants:	Process:
In the figure	AB, 4cm, BC， DE, CD, 8cm, and FG, 9cm	“=” encodes the relationship process of “equate(s)”. This feature is replicated five times

Find the perimeter of ΔAEH

Participants:	Process:
(You) which is implicitly marked, the perimeter of ΔAEH	Find

Figure 5.32: Identifying the experiential features of the statement

The statement is composed of two separate sentences. For the convenience of analysis, “In the figure, AB = 4cm, BC = DE = CD = 8cm, and FG = 9 cm” is labelled as the first sentence while “Find the perimeter of ΔAEH” is labelled as the second sentence.

In the first setence, the circumstance, *in the figure,* has been foregrounded in the beginning of the statement. Participants can be categorized into two classes: linguistic resources signalled by two captical letters: i.e: *AB* and a number with the unit of measure i.e: *4cm*. A list of participants are “*AB”, “BC”, “DE”, “CD”,* and *“FG*” categorized as linguistic resources and *“4cm”, “8cm”,* and *“9cm”* categorized as a number adhered by the unit of measure. Processes in the first sentence are relational processes through encoding the relational process “equate(s)”, into the mathematical symbol “=”. The second sentence is a command, in which the participant, “you” as the actor, has been presumed as a Subject. The process is a material process indicated by the verbal group “find”. The other participant is “the perimeter of ΔAEH”; a nominal group forms the “Head-Thing” structure with the thing being a symbolic expression indicated by the symbol “Δ”.

5.4.2 The ideational commitment in the geometric figure as the second component

The second component within the pedagogic item is a mathematical geometric image. This geometric image is displayed in Figure 5.33.

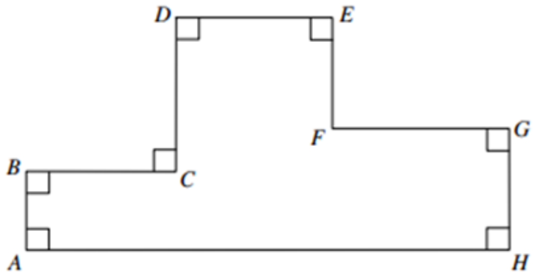
Geometric Image	

Figure 5.33: The geometric image

The geometric image found in Figure 5.33 is an irregular geometric image with functional elements conditioned within this image with the help of “geometrical displays” (O’Halloran, 2005, p. 135). From an ideational perspective, these functional elements are visualised as the lines, the corners, and the points. The “perceptual reality” (O’Halloran, 2005, p. 135) between these functional elements is configured and conditioned in the geometric image. Table 5.5 provides an explanation of the functional elements configured in this geometric image.

Table 5.5: Functional elements identified in Figure 5.33

Functional Elements	Items identified
Points	A; B; C; D; E; F; G; H
Lines (each line is marked by two letters of the points)	AB; BC; CD; DE; EF; FG; GH; HA
Corners (each corner is marked by three letters, the point of the corner is in the middle of the three letters, the symbol “∡” labels corners)	∡ABC; ∡BCD; ∡CDE; ∡DEF; ∡EFG; ∡FGH; ∡GHA; ∡HAB

Note: Different functional elements are separated with a semi-colon, e.g. “;”.

The meaning-making process of mathematical visual images has been explored primarily from an inter-semiotic perspective where mathematical visual images are produced through the "visualization of lexical and symbolic functional elements" (O'Halloran, 2005, p. 168). From an inter-semiotic perspective, mathematical visual images have the potential to maintain the experiential categories of "process, participant and circumstance" (O'Halloran, 2005, p. 168) which were originally perceived by lexical and symbolic resources. Drawing from the framework of re-instantiation outlined in Chapter Four, the approach undertaken here is to treat the mathematical visual image as a particular instance of the "totality of systems" (Painter et al., 2013, p. 134). In this example, the totality is built upon these three components: the statement, the visual image and the multiple choices. As for the mathematical visual image, "(its) meaning resides in the specific options selected" (Painter, et al. 2013, p. 134). Therefore, the specific options selected in the mathematical visual image need to be considered. In terms of the participants, the functional elements proposed in Table 5.5 are the participants identified in the geometric image, consisting of eight points, eight lines and eight corners. In terms of the property of eight different corners, each corner has been labelled by the right-angle symbol: "∟", indicating the corner to be a right angle (90 degrees). In terms of the circumstance, the irregular geometric image depicts the place where different participants are framed, foregrounding the setting where "perceptual reality" (O'Halloran, 2005, p. 135) between these functional elements are established. In terms of the process, this irregular geometric image appears in the form of a static diagram. In visual images, the major processes are "spatial, temporal and relational with entities in the form of the line segments, circles and curves requires explanation" (O'Halloran, 2005, pp. 43–44). This static diagram is concerned with the spatial process in particular. For example, although not quantified, "a perceptual understanding of spatial relations" between different lines in this irregular geometric image is "formed by the line segments", indicating "the distances between two points" (O'Halloran, 2005, p. 143).

5.4.3 Ideational commitment in multiple choices derived from the existing conditions

The purpose of this pedagogic item is to assess students' perception of the geometrical knowledge. The last component within the pedagogic item renders four possible answers to the assessment task underlined in the statement. This last component is displayed in Figure 5.34.

Multiple Choice answers	A	60 cm
	B	74 cm
	C	150 cm
	D	164 cm

Figure 5.34: Multiple choice answers

A scale of four alternative solutions to the assessment task is provided in Figure 5.34. Only one of the four is the correct answer.

In terms of the semiotic situation in Figure 5.34, four separate lines are sequenced and labelled by capitalised letters: *A, B, C*, and *D*. The alphabetic letters has been encoded sequentially. This quality is the default quality of the Hindu-Arabic numeration system in which Hindu-Arabic numbers, such as 1, 2, 3, and 4 have been encoded with a system of sequencing. This system is transformed from numbers and encoded in letters from the English alphabet. The sequencing indicated in Figure 5.34 is in accordance with the sequence of letters appearing in the English alphabetical table, following an order of A, B, C and D. This sequencing echoes the progression of a verbal text which "unfolds over time in a dynamic, sequential way and language has a rich potential for the construal of temporal deixis, sequencing, location, phasing and aspect" (Painter et al., 2013, p. 133).

The presentation of information in each line is achieved through a recurrent pattern in the form of "Label^Number^Unit". The symbol "^", borrowed from the convention developed by Halliday (1994), could be verbalised as "is followed by". Experiential potential has been packaging in each line. In terms of the content of each line, each of them is an alternative answer to the assessment question. One of the four alternatives is the correct answer. With respect to the nature of the correct answer, the correct answer in its own right commits the ideational content specific to the assessment task that has been conditioned in this pedagogic item. This correct answer is derived from the other two components: the linguistic statement and the visual image. This derivation is achieved through a constant dialogue between the mathematical knowledge to be tested and the semiotic resources available.

Based on the semiotic situation in Figure 5.34, the experiential content for the correct answer is associated with the participant that needs to be derived, based on the other two components. Treating this component as a whole, multiple choices commit the relevant ideational meaning on the one hand, and on the other hand, they commit irrelevant ideational meaning as well. The judgement with regard to which possible answer is correct relies on the combined effort committed by the other two components. Table 5.6 displays the functional elements identified in Figure 5.34.

Table 5.6: Functional elements identified in Figure 5.34

Functional Elements	Options
Participants	A 60 cm
	B 74 cm
	C 150 cm
	D 164 cm

It must be noted here that based on the nature of the pedagogic item, only one of the four participants in Table 5.6 could be derived from the other two components. The remaining three participants commit the ideational meaning irrelevant to the pedagogic item.

5.4.4 Comparison between ideational commitments: Introduction

Building upon the comparison between visual image and verbal language in children' picture books, the bimodal texts are considered by Painter and her colleagues (Painter et al., 2013). In their work, the instantiation of meaning potential has been identified in different semiotic systems. Extending their understanding, that “a bimodal text has the potential to commit greater or lesser amounts of any kind of meaning from either semiotic system”, each of the three components identified in the pedagogic item “has the potential to commit greater or lesser amounts of any kind of meaning” (Painter et al., 2013, p. 149) compared to the other two components. The emphasis of the comparison of different commitments of meaning has been placed onto the Ideational meaning as outlined in Chapter Four. This approach has been applied from Sections 5.4.1 to 5.4.3 for the understanding of how the experiential content in each component has been instantiated. These experiential contents have been synthesized in Table 5.7.

Table 5.7: Experiential contents in the statement

		Participants		Process	Circumstance
Statement	Sentence 1	Lines:	AB, BC，DE, CD, and FG,	Relational Process is encoded as mathematical symbolism of “=”	In the figure
		Numbers:	4cm, 8cm and 9cm		
	Sentence 2	(You) which is the actor is omitted		Material process of “find”	Not mentioned
		the perimeter of ΔAEH			
the geometric image		Points	A; B; C; D; E; F; G; H	Spatial process	An irregular geometric image functions as the setting
		Lines	AB; BC; CD; DE; EF; FG; GH; HA		
		Corners	∠ABC; ∠BCD; ∠CDE; ∠DEF; ∠EFG; ∠FGH; ∠GHA; ∠HAB		
the multiple choices		A. 60 cm		Process is omitted	Circumstance is omitted
		B. 74 cm			
		C. 150 cm			
		D. 164 cm			

Since each component has its own way of making meaning, "there is not always a tidy complementarity" (Painter et al., 2013, p. 136) between these components. As could be seen from Table 5.7, in terms of the ideational commitment, each component has its own emphasis. With reference to the comparison between different commitments of ideational meaning, the first category to look at is how the participants have been instantited in different components and how these participants are connected based on the framework outlined in Chapter Four.

5.4.5 Comparison between ideational commitments: Participants

Participants in the statements could be categorized into four types. In the first sentence, the participants are the labels for the lines (e.g. AB) and the length as shown in number plus unit (e.g. 4cm). In the second sentence, the participants are the Actor (you) mentioned implicitly in the command, and the Goal: "the perimeter of ΔAEH". The semiotic resources applied to manifest the participants in the statements are verbal language and mathematical symbolism intersecting with each other. As for verbal language, three sub-categories are identified in the statement, namely, capitalised English alphabetical letters (e.g. AB), Hindu-Arabic numbers plus a unit of measure (e.g. 4cm); and nominal groups, namely, "the perimeter of". Mathematical symbols (e.g. Δ) co-exist with the nominal group (e.g. the perimeter of) and capitalised English alphabetic letters (e.g. AEH) to compose a nominal group*: the perimeter of ΔAEH*. The "ΔAEH" stands for the triangle AEH with A, E, and H being the three points of that triangle. Table 5.8 includes the list of participants committed in the statement.

Table 5.8: List of participants in the statement

The component being examined	**Sentence Number**	**Participants**	
Statement	Sentence 1	Lines:	AB, BC，DE, CD, and FG,
		Numbers:	4cm, 8cm and 9cm
	Sentence 2	"You" is the actor and "you" is omitted	
		the perimeter of ΔAEH	

As could be inferred from Table 5.8, the participants in the statement are mostly related to the field of geometry (trigonometry in particular). The only participant that is not associated with geometry is the Actor (you) in the command (*"Find the perimeter of ΔAEH"*), that actor having been only implicitly mentioned.

The ideational meaning commitment for the visual image has been elaborated in Section 5.4.2. Here, the criteria for identifying the visual participants in geometric images are stated. As has been articulated by O'Halloran (2005), the participant realised in the geometric image is through the visualisation. With respect to the possible choices available in geometric images, "the major visual participants are lines, line segments, circles, arcs and curves and geometrical shapes which are the visual representations of the relations" (O'Halloran, 2005, p. 56). O'Halloran's (2005) list provides the possible choices that could perform as visual participants in the geometric images through enumeration: to enumerate the possible candidates for visual participants. Based on the visual geometric image in this pedagogic item, visual participants include the three major types, namely the points, lines and corners, all of which are labelled by capitalised letters. With respect to the property of corners, a supplementary mathematical symbolism in the shape of "∟" has been attached to every corner on the geometric image. This use of mathematical symbol implies that corners marked by "∟" are right angles, specifying the property of each angle. A list of visual participants identified in the geometric image is presented in Table 5.9. The properties in the other two types of participants have not been specified in the visual image.

Table 5.9: List of visual participants in the geometric image

The component being examined	**Visual participants**	**Labels**
the geometric image	Points	A; B; C; D; E; F; G; H;
	Lines	AB; BC; CD; DE; EF; FG; GH; HA
	Corners	∠ABC; ∠BCD; ∠CDE; ∠DEF; ∠EFG; ∠FGH; ∠GHA; ∠HAB As marked by "∟", these corners are right angles

At this point, the dialogical relationship between the statement and the geometric image could be argued from the perspective of their different ideational commitments with respect to their different coverage of participants. Drawing from the research framework provided by Hood (2008) and Painter et al. (2013), the relationship could be categorised as direct translation, informing, generalisation, investment, and encoding.

First, there is a direct translation between the participants in the statement and the visual participants in the geometric image. Lines in the statement that are presented as two capitalized letters have been translated into the visualized lines labelled by two capitalized letters. That is to say, lines identified in the statement correspond to visualised lines in the geometric image. For example, correspondence could be identified between "AB, BC, CD, DE, and FG" in the statement and "AB, BC， CD, DE, and FG" in the geometric image. Figure 5.35 indicates this direct translation relationship between these two components with the lines in the statement being underlined and bridged with the corresponding lines in the figure with the help of arrows.

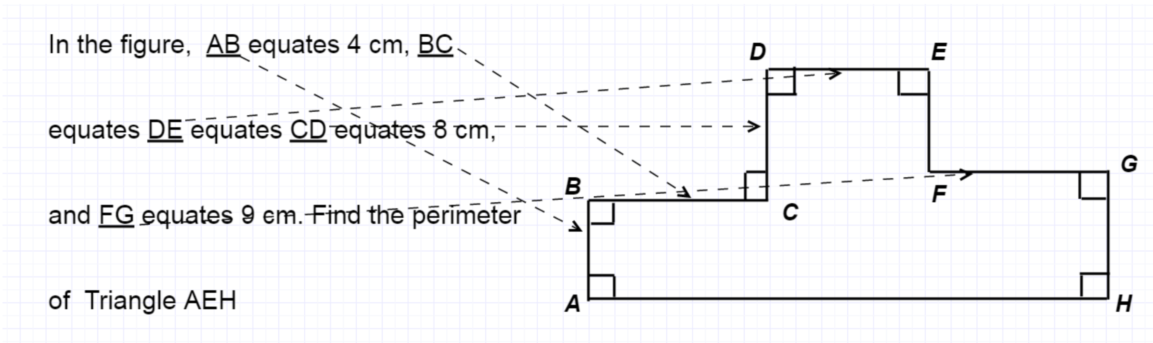


Figure 5.35: The direct translation relationship between these two components

The other type of participant in the statement is constructed through Hindu-Arabic numbers (e.g. 4), adhering with a unit of measure (e.g. cm). In the linguistic statement, this type of participant is the Attribute of its Carrier linked by the relational process (Halliday & Matthiessen, 2004, p. 236). For example, "4cm" is the Attribute of its Carrier: AB, assigning the property of "measure" (Halliday and Matthiessen, 2004, p. 236) onto the Carrier. In this example, the type of measure carried by the Attributes (4cm, 8cm and 9cm) in the statement is concerned with length: a quantity of distance. Therefore, each line in the statement has been quantified with a property of physical distance in the two-dimensional space.

Following the attribution, the property of each line in the visual image is therefore enriched. This enriched version is provided in Figure 5.36 with the property of length being attributed.

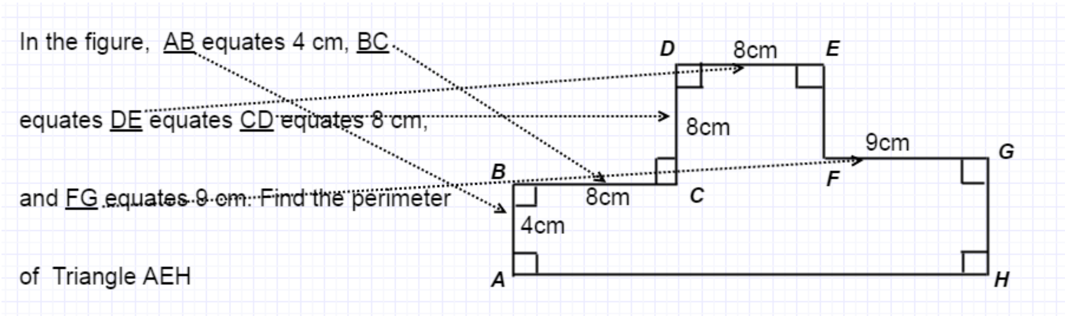


Figure 5.36: Property of length has been attributed to lines in the visual image

With respect to the dialogical relationship between the statement and the visual image, the function of the participants displayed as Hindu-Arabic numbers adhered with a unit of measure in the statement, is to attribute the property of length to the physical lines in the geometric images, enriching the lines in the geometric images with the physical property of length. This physical property which has not been instantiated in the original visual images is inferred based on the Attributes (such as 4cm) as participants explicated in the statement. This relationship is termed here as an attribution through which the properties of the visual participants are enriched by the Attributes underlined in the statement.

The dialogical relationship between the statement and the visual image could also be investigated based on a special participant in the statement. That participant in the statement is informative of a hidden participant in the visual image. This relationship is termed as informing through which the participants implied in the visual image are unravelled with the assistance of correspondent participants in the statement.

To be more explicit, "the perimeter of ΔAEH" has been explicated in the statement. This participant is the Goal of the material process "find" in the command *"Find the perimeter of ΔAEH"*. However, this participant has not been indicated in the visual image. The participant in the statement appears as a nominal group ("the perimeter of ΔAEH") in which the logical relationship between different nominals (*the perimeter* and *ΔAEH*) could be identified. In this nominal group, the preposition "of" is "the generalized marker of a structural relationship between nominals" (Halliday & Matthiessen, 2004, p. 333). "The perimeter" is the Head while "ΔAEH" is the Thing. The relationship between "the perimeter" and "ΔAEH" resembles the "partitive" relationship underlined by Halliday and Matthiessen with "the perimeter" representing a "portion" (Halliday & Matthiessen, 2004, p. 333) of the thing "ΔAEH". The participant "*the perimeter of ΔAEH*" in the statement is reflective of two layers of instantiation: the instantiation of the Thing: "ΔAEH" and the instantiation of the Head "the perimeter" of that Thing. The Thing "ΔAEH" as one layer of instantiation could be visualised in the geometric visual image informing a newly drawn triangle in the visual image.

Figure 5.37 converts the original geometric image with the addition of two auxiliary lines (*AE* and *EH*). The instance of semiotic complex: "ΔAEH" in the statement has been converted into an explicit triangle (Triangle AEH) in the visual image. The mathematical symbolism "Δ" has been converted into the visual image of a triangle, and the three capitalised letters "AEH" stand as the three different points of that triangle, labelling and naming this triangle. For ease of visibility, these two auxiliary lines are marked with dotted lines, distinct from other lines that are solid lines in the visual image.

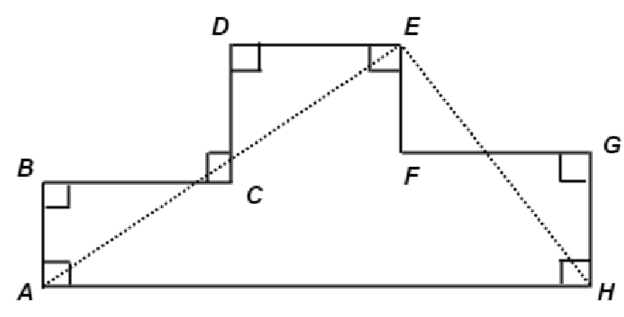
Converted Geometric Image	

Figure 5.37: The converted geometric image

Figure 5.37 indicates how the Thing "ΔAEH" in the statement informs the addition of a triangle in the visual image. Triangle AEH thus emerges from the existing geometric image, fulfilling the first layer of recontextualisation. In this example, the informing is achieved through de-contextualising "ΔAEH" in the statement from its original setting and recontextualising this participant into the visual image, adding an auxiliary participant that was not indicated in the original semiotic construction of the visual image.

The Head "the perimeter" has not been indicated in Figure 5.37. "The perimeter" which is the Head of that participant indicates the length of the boundary of Triangle AEH. One of the characteristics of "the perimeter" is its quantifiablity; that is, its state of being quantifiable. The quantity needs to be calculated, as it could not be discerned directly from the informed Triangle AEH in Figure 5.37, though it offers a basis from which the quantity could be inferred and calculated. This stepping-stone is concerned with the emergence of the Triangle AEH whose boundary length needs to be quantified.

Therefore, the relationship of informing tells how the Thing "ΔAEH" is capable of informing the emergence of a newly drawn triangle in the visual image. In terms of the quantifiable perimeter, "the perimeter of ΔAEH" which is the other layer of instantiation in the statement could not been instantiated directly in the visual image. It is the Goal of the material process within the statement, required to be figured out with the assistance of relevant mathematical knowledge.

In this example, both attribution and informing are initiated by the statement. The recontextualisation operates in a direction from the statement to the visual image, enriching one line in the visual image with a quantifiable property and informing a new participant; that is, a new triangle in the visual image. Conversely, the visual image could also initiate recontextualisation, committing the participants that have not been mentioned in the statement. For example, after the comparison between different participants in the statement and those in the visual image, additional participants in the visual image could be found. These additional participants include the three lines: EF; GH; HA, and eight points: A; B; C; D; E; F; G; H and eight corners; ∠ABC; ∠BCD; ∠CDE; ∠DEF; ∠EFG; ∠FGH; ∠GHA; ∠HAB. Therefore, generalisation between the statement and the visual image could be identified with the geometric image in this pedagogic item and commits different participants that are not indicated in the statement. This difference is indicated through the nature of the geometric image. Geometric visual images convey a visualisation of "functional elements" (O'Halloran, 2005, p. 177) that have not been mentioned in other components. What can be inferred from this relationship is that participants related to lines in the statement are part of the participants in the visual image, forming a meronymic relationship between them. As for the rest of the participants depicted in the visual image, they have the potential to be utilised as the clue to quantifying the Goal: "the perimeter of ΔAEH".

"The perimeter of ΔAEH" which is Goal of the material process is quantifiable. Possible answer matches with "the perimeter of ΔAEH" have been blended with three irrelevant answers in the last component of the pedagogic item. With reference to the ideational commitment of the multiple choices outlined in Section 5.4.3, only one of the four possible choices: A. 60 cm, B. 74 cm, C. 150 cm, D. 164 cm is the correct answer to the assessment, committing the correct ideational meaning. The rest of the possible choices are irrelevant, misleading the students. With respect to the comparison between different components in terms of the commitments of participants, the correct answer is a specification of the Goal "the perimeter of ΔAEH" in the statement, demonstrating a one-to-one relationship. With respect to the nature of the participants in this component, each of the possible choices has been quantified in the form of a Hindu-Arabic number followed by a unit of measure.

5.4.6 Comparison between ideational commitments: Process and Circumstance

This section is concerned with how the commitments of processes in different components interact with each other. Among the three major experiential elements in a clause (participant, process and circumstance), "process is the most central element" (Halliday & Matthiessen, 2004, p. 176). The nature of participants "vary according to the type of process" (Halliday & Matthiessen, 2004, p. 176) while "circumstantial elements are almost always optional augmentations of the clause rather than obligatory components" (Halliday & Matthiessen, 2004, p. 175).

In the statement, the first type of process is a relational process indicated through mathematical symbolism "=" in the first sentence. The first sentence could be segmented into three independent clauses: "AB = 4 cm", "BC = DE = CD = 8 cm" "and FG = 9 cm" with each clause in its own right consisting of its participants and process(es). The default relational process is realised in the pattern of Carrier ^ Relational Process ^ Attribute such as "AB = 4 cm" and "FG = 9 cm" when the mathematical symbol "=" could be verbalised as "equals" signifying the relational process. This default structure echoes with the configuration of experiential contents in verbal language where "every experiential type of clause has at least one participant" and "participants are inherent in the process" (Halliday & Matthiessen, 2004, p. 175).

In the first sentence, a complex mathematical equation: "BC = DE = CD = 8cm" could also be found. This complexity is specific to scientific discourse such as in mathematics, physics and so on. A mathematical equation consists of "strings for Operative processes and participants" (O'Halloran, 2005, p. 111). These strings are "repeated, substituted, re-organized and simplified according to mathematical definitions, algebraic laws, and other established results for algebraic operations" (O'Halloran, 2005, p. 111). The strings for an operative process could be extended to include a relational process as well. In this example, relationship process "=" has been repeated three times, connecting different participants to encode their underlying relationship with the help of the mathematical symbol "=". The notion of "*multiple participants"* (O'Halloran, 2005, p. 106, italicised in the original version), indicating that participants in symbolic equations "appear to play equally key roles" (O'Halloran, 2005, p. 106), could be expanded to include the relational process. The first three participants: "BC", "DE" and "CD" belong to the same category, and play the same role in the series of relational processes.

A different category of participant emerges at the end of the equation. The participant that was elaborated in the last section is concerned with the quantifiable property of length indicated through the Hindu-Arabic number and the unit (e.g. 8cm). In terms of the delicate relationship within this equation, each line marked with the capitalised letter is the carrier and the Hindu-Arabic number and the unit together compose the attribute. Therefore, this string of relational processes and participants could be converted into Table 5.10 indicating the experiential category within this equation.

Table 5.10: Experiential category within the equation

Equation	**BC**	**=**	**DE**	**=**	**CD**	**=**	**8cm**
Experiential Category	Carrier	relational process	Carrier	relational process	Carrier	relational process	Attribu te

The string of participant and process is the typical characteristic of a mathematical equation. In this manner of information construction, the relationship between carrier and attribute is established not only through linguistic resources but also through encoding the relational processes into mathematical symbolism. The relationship between different carriers and attributes in this equation is established based on the algebraic law of equation. The law of the equation is the most basic and default characteristic of mathematics, suggesting that variables bridged by the equation symbol "=" are equal. Based on the law of the equation, carriers are empowered with the physical property of length indicated through the attribute.

As has been mentioned in the previous section, the attribute in the statement could be empowered with the experiential content in the visual image. The relationship between carriers and attributes are established through the means of a relational process such as the mathematical symbol "=". The relational process in the statement is omitted in the visual image, indicating "a loss of meaning" (O'Halloran, 2005, p. 134) in the dialogue between the statement and the visual image, where the property for each line in the visual image is dependent on the property in the statement. The statement commits the relational processes. These relational processes link the carrier with the attribute. The attribute could thus be bridged with relevant participants in the visual image, enriching them with quantifiable length.

The process in the second sentence is a material process "find". The goal of the material process "find" is "the perimeter of ΔAEH". In the statement, the goal as the participant is inherent to the material process "find". When the goal becomes quantifiable, the quantity (length in this example) has been embedded in the multiple-choice alternatives. Table 5.11 combines the statement and the multiple choices in the search for the correct answer.

Table 5.11: Bridge the statement with multiple choices

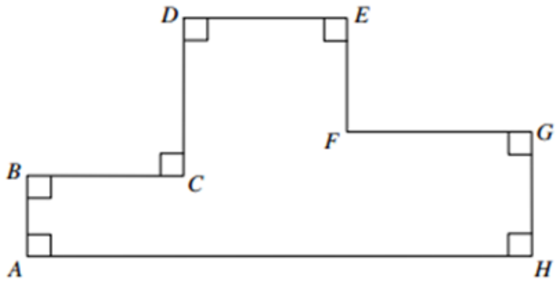
Semiotic resources and Experiential Categories	**Statement**			**Multiple Choices**
Semiotic resources	(You)	Find	the perimeter of ΔAEH	A. 60 cm B. 74 cm C. 150 cm D. 164 cm
Experiential Categories	Actor (omitted)	Material Process	Goal The Goal is quantifiable	Four Participants as Quantity One of the quantity is the quantifiable counterpart of the Goal in the statement

Based on Table 5.11, there is a transformative relationship between the goal in the statement and the participants in the multiple-choice answers. The goal in the statement has been instantiated as a quantifiable unit whilst the four participants in the multiple-choice answers are all concerned with quantity. Only one of them is the counterpart of the goal in the statement. The material process "find" in the statement directs the transformation from the quantifiable nature of "the perimeter of ΔAEH" to the real quantity that is composed of numbers plus a unit of measure i.e.: 60cm, 74cm, 150cm or 164cm. The multiple-choice process is omitted. The multiple choices could be rendered back into a complete clause echoing with the assessment to form up as a "Question-Answer" pattern. For example, 60cm could be rendered back as *"The perimeter of ΔAEH is found to be60cm"* indicating a linguistically comprehensive expression. Similarly, with the other three alternatives, their linguistically comprehensive versions being: *"The* *perimeter of ΔAEH is found to be 74cm", "The perimeter of ΔAEH is found to be 150cm"* and *"The perimeter of ΔAEH is found as 164cm".* However, these expressions have been contracted into nominal groups "60cm", "74cm", "150cm" and "164cm". These nominal groups are meaningful enough in the sense that transformation between "the perimeter of ΔAEH" and four quantifiable answers has been achieved with the assistance of the material process: "find". Therefore, the material process with regard to the multiple choices could be eclipsed.

The material process is also omitted in the visual image due to the static nature of the visual image. The participant triggers the informing of the new triangle AEH in the visual image, as has been considered in the previous section: "the perimeter of ΔAEH". This participant is inherent to the material process "find". Therefore, the material process "find" also results in the emergence of the new participant in the visual image. That is to say, the process in the statement results in the emergence and selection of participants in other components.

With reference to the visual image, it also commits the process of "spatial relation" (O'Halloran, 2005, p. 111). This spatial relational process is realised through textual structure by considering the spatial relationship between different corners, different lines and different points in the visual image. For example, different lines are perpendicular, crossed, or paralleled with the assistance of the spatial arrangements between these different lines. These types of spatial relations are typically found in geometric visual images. These relationships could also be utilized to infer how the participants in other components are realised. For example, in the statement: "*in the figure, AB = 4 cm, BC = DE = CD =8 cm, and FG=9 cm*", the relationships between five carriers (AB, BC, ED, CD and FG) have only been specified in terms of their quantity, let alone their spatial relationship. Their spatial relationships could only be inferred based on spatial arrangements between different lines in the geometric visual image. These spatial relationships are listed in Table 5.12.

Table 5.12: Types of spatial relationships between different lines in the visual image

Enumeration of the relationship between different lines as participants in the statement	Elaboration of the spatial relationships in the visual image
In the figure, AB = 4 cm, BC = DE = CD = 8 cm, and FG=9 cm.	
Between AB and BC	AB and BC are perpendicular to each other, meeting at the initial point B. The initial point "B" and the perpendicular symbol "∟" suggest their relationship
Between AB and DE	AB and DE are perpendicular to each other. Although they do not have an initial point in the given visual image, based on the two-dimensional organization in this irregular image, the perpendicular relationship could be inferred.
Between AB and CD	AB and CD are parallel to each other. Their parallelization is achieved through the two-dimensional organisation through which their spatial relationship is inferred.
Between AB and FG	AB and FG are perpendicular to each other. Although they do not have an initial point in the given visual image, based on the two-dimensional organization in this irregular image, the perpendicular relationship could be inferred.
Between BC and DE	BC and DE are parallel to each other. Their parallelization is achieved through the two-dimensional organisation through which their spatial relationship is inferred.
Between BC and CD	BC and CD are perpendicular to each other, meeting at the initial point of C. The initial point "C" and the perpendicular symbol "∟" suggest their relationship
Between BC and FG	BC and FG are parallel to each other. Their parallelization is achieved through the two-dimensional organisation through which their spatial relationship is inferred.

Enumeration of the relationship between different lines as participants in the statement	Elaboration of the spatial relationships in the visual image
Between DE and CD	DE and CD are perpendicular to each other, meeting at the initial point of D. The initial point "D" and the perpendicular symbol "∟" suggest their relationship
Between DE and FG	DE and FG are parallel to each other. Their parallelization is achieved through the two-dimensional organisation through which their spatial relationship is inferred.
Between CD and FG	CD and FG are perpendicular to each other. Although they do not have an initial point in the given visual image, based on the two-dimensional organization in this irregular image, the perpendicular relationship could be inferred.

Table 5.12 enumerates the possible spatial relationships that could be identified between the five participants: AB, BC, DE, CD and FG in the statements. These participants are associated with the lines in the visual image. The enumeration of the ten pairs of spatial relationships suggests the potential of visual imagery in highlighting spatial relationships between different lines. These spatial relationships were not provided in the statement but could be identified in the visual image based on the two-dimensional construction of the visual image, the positions of the lines and the symbol for a right-angle (∟). Therefore, the visual image has the potential to commit spatial relationships, which have not yet been verbalised in the statement.

It must be noted here that Table 5.12 is only concerned with the spatial relationship between different lines. Once the concern is extended to include all lines, all points and all corners, the visual image will commit numerous pairs of spatial relationships because "visual display in mathematics is more intuitive than the symbolic descriptions" (O'Halloran, 2005, p. 145). Once there are cases of lines, curves, points and corners included as the participants in the visual image, numerous pairs of spatial relationships may be embedded. Therefore, as remarked by O'Halloran (2005, p. 145) "the experiential meaning encoded within the visual display is complex" and it will take much effort to exhaust all the possible pairs of spatial relationships in the geometric visual display. The irregular visual image considered in this pedagogic item is an example revealing the complexity of visual imagery in geometry.

Regarding the commitments of processes, each component has its own emphasis. The relational processes ("=") in the statements are for the purpose of indication: to assign the property to each line. The material process informs the Goal of the Actor. Interpersonally speaking, this command ("*Find the perimeter of ΔAEH"*) is an unmarked choice in requiring information from students during the assessment. Spatial process in the visual image is suggestive of the spatial arrangements of different lines with respect to how they are connected, paralleled and crossed, for example.

Regarding how the circumstantial content has been realised, the priority is to consider the role played by circumstance in construing the experiential content of circumstance in the statement.

The statement starts with the circumstance, "In the figure", focusing on the experiential content of location. From a textual perspective within this prepositional phrase, the use of Deictic: "the" specifies the referent to which the circumstance is directed. From an intertextual perspective, the visual image is thus associated with the circumstance. Compared with participants and processes, the unmarked nature of circumstance is that circumstance occupies a rather peripheral position compared with the other two experiential categories. In this example, however, the circumstance is highlighted in the sense that the primary link between the statement and the visual image is bridged by using the circumstance. The significance of circumstance could also be proved through a textual perspective at the clause level. "In the figure" occupies a Theme position, rendering the first experiential content in the clause. This marked choice of Theme signifies the significance of circumstance in this clause in introducing the assessment task.

With reference to how the circumstance has been activated in the visual image, the irregular image in its own right foregrounds the place in which different participants (lines, corners, points, and triangles) interact with each other. It is a concrete visualisation of the circumstance: "in the figure", narrowing the scope of different figures into only one figure and specifying the exact example regarding the circumstance.

Regarding the different commitments of ideational meaning with reference to circumstance, the relationship between different circumstances in the statement and in the visual image is a typical incident of generalization with the irregular visual image being a kind of geometrical "figure". The verbalised version in the statement is relatively broad. The broadness requires narrowing to a certain type of figure. This specification is achieved through the provision of the visualised geometrical figure as has been offered in this pedagogic item.

After describing and comparing the difference existing in the commitment of experiential meaning, the next task is to see how the knowledge of Pythagoras' Theorem has been practised in these three components, and with reference to how Pythagoras' Theorem has been instantiated in each of the three components. An investigation unfolds with reference to what degree the representation of Pythagoras' Theorem activates and utilises "the experiential content" (Halliday and Martin, 1993, p. 101) in selected data. Section 5.5.7 is dedicated to this discussion.

5.4.7 Pythagoras' Theorem

Pythagoras' Theorem has been prepared as the required mathematical concept based on this the assessment task. Not a single linguistic clue in this pedagogic item has disclosed that it is associated with Pythagoras' Theorem. The clues indicating Pythagoras' Theorem lie in the visual and symbolic resources such as the lines in the triangle AEH, the right triangle marked by the symbol "∟" in the irregular geometrical image, and the quantifiable nature of the perimeter.

The degree to which Pythagoras' Theorem has been associated with this pedagogic item is required to be compared with the place where Pythagoras' Theorem as a totality is preserved. This place will be "Theorem" in the textbook. The next section will be concerned with how Pythagoras' Theorem has been introduced in the textbook with particular regard for how the commitment in that pedagogic item has contributed to the totality of the representation of Pythagoras' Theorem. As well, attention will be directed to the knowledge representation of other pedagogic items, including one being considered in this section with the assistance of different mechanisms of mathematical cues. Because the pedagogic item to be considered in next section is the Theorem, the totality of the experiential meaning concerned with Pythagoras' Theorem needs to be preserved.

A comparison between the pedagogic item considered in this section and the pedagogic item to be considered in the textbook will be rendered in the summary of this chapter, offering a complementary perspective in viewing how the knowledge of Pythagoras' Theorem has been preserved in these two pedagogic items.

5.5 Ideational commitment in the textbook

The analysis of the assessment in the previous section (Section 5.4) foreshadows the analysis in this section. The purpose of this section is to elaborate how Pythagoras' Theorem has been instantiated in the textbook (Wong & Wong, 2007) by focusing on one pedagogic item where the totality of Pythagoras' Theorem has been instantiated. The discussion starts from an overview of the semiotic situation concerned with this pedagogic item before moving onto the discussion on how each component within this pedagogic item are co-related with each other. This section also contributes to an understanding of how the ideational meaning within each component has been committed.

5.5.1 Introduction to the pedagogic item: The instance in the textbook regarding Pythagoras' Theorem

The pedagogic item under investigation is presented in Figure 5.38.

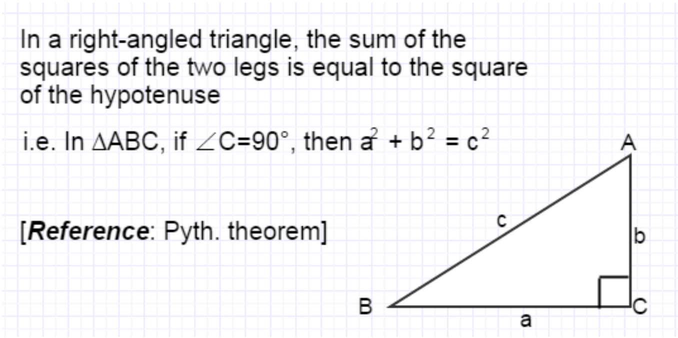


Figure 5.38: Pedagogic item in association with Pythagoras' Theorem in the textbook (Wong & Wong, 2009, p. 103)

The analysis in this section focuses on this pedagogic item (Wong & Wong, 2009, p. 103). The complexity of this instance involves both the range of semiotic construction and the whole package of the knowledge structure. In terms of the knowledge construction, Pythagoras' Theorem as a mathematical concept has been preserved comprehensively. By saying comprehensively, the justification corresponds with mathematical knowledge structure described in Chapter Two. To reiterate, one mathematical concept could subsume other mathematical concepts and could be subsumed by other mathematical concepts. For the sake of the analysis, this pedagogic item in Figure 5.38 is the place where the whole package of Pythagoras' Theorem is included. In terms of the semiotic construction, the meaning making process in the selected pedagogic item is fruitful due to the use of mathematical symbolism, visual image in line with linguistic resources.

5.5.2 Generic structuring of this pedagogic item

Figure 5.38 is identified as one pedagogic item, subsuming a series of components in exemplifying Pythagoras' Theorem. This pedagogic item is composed of four components: a linguistic definition of Pythagoras' Theorem, a specific example of Pythagoras' Theorem in symbolic equations, a visual image in a geometric diagram and a coda providing the name of this theorem. Each component is an individual instance of the instantiation of Pythagoras' Theorem, varying in their different ways of instantiating the knowledge.

The arrangements of these components are aligned with O'Halloran's (2007b) notion of "Juxtaposition and Spatiality" through which four textually differentiated components within the pedagogic item in Figure 5.38 could be separated. These components which are spatially aligned could be argued from a genre perspective because their structuring is realised by the staging (Martin & Rose, 2013, p. 234) of the pedagogic item.

Figure 5.39 is re-formed from the generic structuring provided in Figure 5.38, highlighting the flow of information taking place between these individual components within a pedagogic item.

<table>
<tr><th>Different Stages</th><th colspan="2">Generic Structure</th><th>Function of Each Stage</th><th>Semiotic Resources</th></tr>
<tr><td>Stage One</td><td colspan="2">Statement</td><td>The definition of the theorem</td><td>Language</td></tr>
<tr><td rowspan="2">Stage Two</td><td colspan="2">Specification</td><td rowspan="2">Specification of the theorem through the provision of a contextual specific example</td><td rowspan="2">Language,

mathematical symbolism, and visual image</td></tr>
<tr><td>Symbolic equation</td><td>Visual diagram recontextualises the participants and circumstances in Symbolic equation</td></tr>
<tr><td>Stage Three</td><td colspan="2">Coda</td><td>Summary of the theorem through the provision of a terminology</td><td>verbal language</td></tr>
</table>

Figure 5.39: The generic structuring provided by the Theorem

This type of mini-genre is termed a *Theorem* whose generic structuring is indicated as Statement ^ Specification ^ Coda. The symbol "^" which is a traditional convention adopted in SFL, could be verbalized as "is followed by", suggesting the sequence between different stages.

The imposing of a genre perspective onto this specific pedagogic item is from the perspective of highlighting the importance of knowledge progression when different components have been staged. The interpretation could therefore be following a sequential order through which the meaning flow within this pedagogic item could be understood. This perspective is also intended to argue that this manner of information flow, Statement ^ Specification ^ Coda, is recurrently occurring within a series of similar pedagogic items, following similar generic structuring exemplified in this pedagogic item. The generic structuring in this pedagogic item also confirms the importance of the spatial arrangements within this pedagogic item.

From Sections 5.5.3 to 5.5.5, the discussion is concentrated on the specific generic stages composed by different components. Moving from the overall generic structuring of the pedagogic item to how Pythagoras' Theorem is embedded within each component, the work requires an explanation of how knowledge is recontextualised within and across different generic stages. The representation of Pythagoras' Theorem in this pedagogic item unfolds from the Statement, followed by the Specification and resides in the Coda. It also involves a multi-semiotic perspective by using a complete range of mathematical semiotic resources: verbal language, mathematical symbolism and visual images.

5.5.3 Knowledge progression: to begin with the Statement

The commencing statement is presented in Figure 5.40.

	Generic Stage	Representation
Stage One	Statement	In a right-angled triangle, the sum of the squares of the two legs is equal to the square of the hypotenuse.

Figure 5.40: The definition of Pythagoras' Theorem in Wong and Wong (2009, p.103)

In terms of the social function of the Statement, it is the working definition of Pythagoras' Theorem provided by Wong and Wong (2009, p. 103). This definition presented in verbal language is the place where Pythagoras' Theorem is firstly instantiated in the pedagogic discourse of the textbook. Since the semiotic resources associated with this statement are verbal language, the IDEATION system in Martin and Rose (2003, 2007 & 2014) is adopted here.

The IDEATION system (Martin 2002, Martin & Rose, 2003, 2007 and 2014) is proposed as a means to understand the Field of discourse at discourse semantic level where the representation of knowledge could be discussed (Martin & Rose, 2014, p. 75). IDEATION interprets the internal relationship between different lexical items in the unfolding of text at both clause level and discourse level with the help of three complementary systems: Taxonomic Relation, Nuclear Relation and Activity Sequence. A central similarity between these three systems is that all of them were intented to account for linguistic resources when they were proposed.

Taxonomic relationship indicates the possible lexical relationship between different lexical items within a text. The expected knowledge and expected knowledge structure will be crucial in understanding the taxonomic relationship between different lexical items (Martin & Rose, 2014, p. 77).

Nuclear Relation determines the lexical relationships between different grammatical categories such as participants, process and circumstances by deciding which lexical items will be more central in formulating the clause. Within nuclear relations, the modifying relationship between adjective and noun is particularly substantial in unveiling the invisible knowledge structure between different technical terms. Two sub-classifications in the modifying relationship, namely 1) Classifier–Thing, and 2) Epithet–Thing, are central in underlining the part-whole relationship and class-member relationship respectively between different technical terms.

Activity sequence addresses the use of lexical items in constructing the temporal relationships between different events. Their temporal relationships, according to Martin and Rose (2014), are "expected by a field" (p. 101).

The decomposing of the statement involves two steps, namely 1) to decompose the nuclearity in the clause; and 2) to decompose the nuclearity within each word group (nominal group and verbal group).

Figure 5.41 displays the nuclearity at clause level, addressing the experiential category that each word group is assigned.

Clause	In a right-angled triangle	the sum of the squares of the two legs	is equal to	the square of the hypotenuse
Nuclearity in the clause	Circumstances	Participant as Agent	Relational Process	Participant as Range
Word groups	prepositional phrase	nominal phrase	verbal group	nominal group

Figure 5.41: Nuclearity of the clause: "In a right-angled triangle, the sum of the squares of the two legs is equal to the square of the hypotenuse."

As could be inferred from Figure 5.41, the Statement have been segmented into different word groups: one prepositional phrase, two nominal groups and one verbal group. Central information extracted from the word groups are rudimentary elements built on which the Statement is established.

Figures 5.42 to 5.45 display the nuclearity in each word group.

Layer 1	Prepositional phrase*: In a right-angled triangle*			
Preposition	*in*			
Layer 2	Nominal phrase			
	a right-angled triangle	a	right-angled	triangle
Central function		non-specific Deictic	Classifier	Thing
word class for the central information				triangle: noun

Figure 5.42: Nuclearity in the prepositional group: "*in a right-angled triangle*"

The word group explored in Figure 5.42 is a prepositional phrase composed of a preposition "*in*" followed by a nominal group "*a right-angled triangle*". This prepositional phrase could be layered into two stages. The central information lies in the second layer where "*triangle*" as the "Thing" has been modified by the Classifier "*right-angled*", and determined by the non-specific Deictic "*a*". The central information extracted from this word group is "*triangle*".

Nominal group: *the sum of the squares of the two legs*			
Layer 1	the sum of	the squares of the two legs	
Central function	Head	Thing	
Layer 2		Nominal groups: the squares of the two legs	
		the squares of	the two legs
Central function		Head	Thing
Word class for the central information	sum: noun	squares: noun	legs: noun

Figure 5.43: Nuclearity in "the sum of the squares of the two legs"

As shown in Figure 5.43, the nominal group: "*the sum of the squares of the two legs*" is composed of a two-layered "Head-Thing" structure, with the second "Head-Thing" structure being embedded in the Thing of the first layer. The central information extracted from this nominal group is composed of a list of nouns: "*sum*", "*squares*" and "*legs*".

Verbal group: *is equal to*

Layer 1	is equal	to
central function	Event	Particle
Word class for the central information	equal: adjective	
Process type	relational process type	

Figure 5.44: Nuclearity in "*is equal to*"

Figure 5.44 is a verbal group indicating a relational process. The central information is "*equal*", addressing a state of equivalence between different entities when connected with this relationship process.

Nominal group: *the square of the hypotenuse*		
Layer 1	the square of	the hypotenuse
Central function	Head	Thing
Word class for the central information	square: noun	hypotenuse: noun

Figure 5.45: Nuclearity in "*the square of the hypotenuse*"

The central information extracted from the nuclearity in Figure 5.45 is concerned with two nouns: "*square*" and "*hypotenuse*" based on a "Head-Thing" structure.

The nuclearity at clause level informs the nuclearity at word group level, which in turn provides the construal of Pythagoras' Theorem with a taxonomic foundation by extracting the central information. That is to say, without any prior knowledge of what Pythagoras' Theorem is about, the nominal groups (*right-angled triangle, sum, leg, square, hypotenuse*) together with the relational process (*is equal to*) are the prerequisite knowledge that Pythagoras' Theorem is established. At a linguistic level, the relationship between Pythagoras' Theorem and these word groups is a taxonomic relationship of "meronym" (Martin & Rose, 2003) with these word groups as parts of Pythagoras' Theorem. As has been justified in previous sections, the meronym at linguistic level could be converted into the hierarchical knowledge structure following Bernstein's (2000) knowledge structures. It is argued here that mathematical knowledge behind these word groups is absorbed by the knowledge of Pythagoras' Theorem through translating the relationship at linguistic level into the relationship at knowledge construction level. This converted model is presented in Figure 5.46.

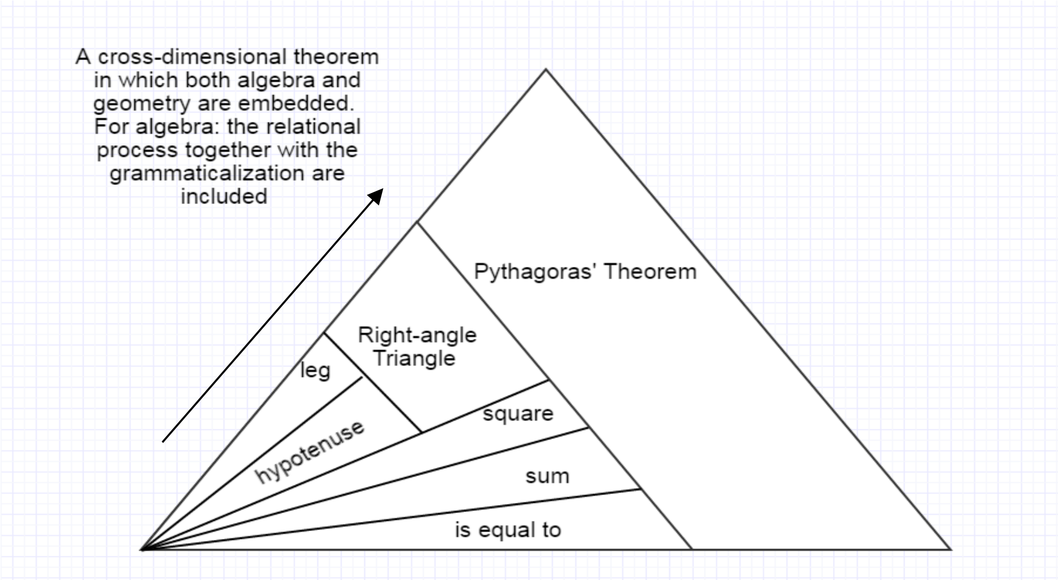


Figure 5.46: A knowledge structure about Pythagoras' Theorem converted from the taxonomic relationship identified in the Statement

In terms of the knowledge structure of Pythagoras' Theorem, it is a cross-dimensional mathematical concept in which algebra and geometry meet with each other. For example, as has been mentioned in Chapter Four, the discovery of Pythagoras' Theorem (or its Ancient China counterpart: Gou Gu Theorem) is in the field of astronomy. With reference to algebra, algebraic laws such as "square", "sum" and "is equal to" are all the basic elements contributing to Pythagoras' Theorem. With reference to geometry, right-angled triangle and the elements within right-angled triangles such as *"leg", "hypotenuse",* and *"right-angle"* contribute to Pythagoras' Theorem. This cross-dimensional nature of Pythagoras' Theorem converted from the taxonomic relationship at linguistic level is an important property outlined in the Statement.

Although Pythagoras' Theorem is not mentioned in the statement, the way that the information is construed within the Statement is through illustrating the prerequisite components of Pythagoras' Theorem, providing nominal groups and then linking these lexical items with a relationship process.

After discussing the knowledge structure of Pythagoras' Theorem instantiated in the Statement, the contextual dependency level of the Statement will be considered. Pythagoras' Theorem is tied within a context of "a right-angled triangle" conditioned by the circumstance "in a right-angled triangle". Therefore, the premise for Pythagoras' Theorem has been conditioned as *a right-angled triangle* without narrowing the exact contextual descriptions on what exactly a right-angle triangle should look like. In reality, the types of right-angled triangle are unlimited. In the Statement, this general applicability enables Pythagoras' Theorem to be defined and become workable under any contexts, as long as the context is conditioned as "a right-angled triangle", since this is the only contextual requirement specified in the Statement.

5.5.4 Knowledge progression: contextual requirements increase at Specification stage

Following the Statement is the generic stage termed Specification. The components within this stage are composed of one mathematical symbolic equation, namely "In ΔABC, if ∠C = 90°, then $a^2 + b^2 = c^2$" (Figure. 5.47), and one geometric diagram (Figure 5.48).

In ΔABC, if ∠C = 90°, then $a^2 + b^2 = c^2$

Figure 5.47: Symbolic equations concerned with Pythagoras' Theorem (adapted from Wong & Wong, 2009, p. 103)

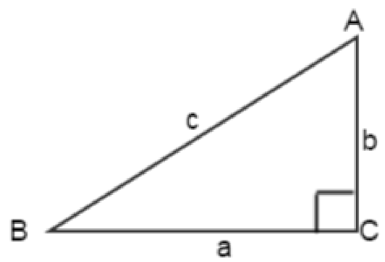


Figure 5.48: A geometric visual image (adapted from Wong & Wong, 2009, p. 103)

These two components are juxtaposed and aligned with each other. This generic stage is termed Specification, because the Pythagoras' Theorem represented at this stage has been tied to a specific context: "In ΔABC, if ∠C = 90°". Experientially speaking, two symbolic expressions "In ΔABC" and "if ∠C = 90°", specify the "circumstances" under which Pythagoras' Theorem is identified in this symbolic equation. The geometric diagram in Figure 5.48 recontextualised the "circumstances" in the symbolic equations by visualizing the "circumstances" in a geometric diagram of a right-angled triangle with the help of mathematical symbolism and visual images.

Compared with the linguistic statement explained in Section 5.5.3, the way in which the information is embedded contains more contextual requirements than those at the Statement stage. At the Statement stage, the contextual requirement is "in a right-angled triangle" while at the Specification stage, the contextual requirements are "In ΔABC" and "if $\angle$C = 90°". The differences between these two contextual requirements lie in their different coverage of the right-angled triangle to which they refer. In the statement, the lexical item used for the circumstance is "in a right-angled triangle". This circumstance is a general requirement and could be applied to any right-angled triangle. Contrasted to the Statement, circumstances in the symbolic equations are "In ΔABC" and "if $\angle$C = 90°". These two symbolic expressions together are reflective of a specific case of a right-angled triangle: triangle ABC with corner C is a right angle. The visual diagram further specifies the contextual requirement in the symbolic equation through a visual provision of the exact shape and size of a right-angled triangle, instantiating the perceptual reality between the lines, points and corners in the visual image.

Therefore, Pythagoras' Theorem as represented at the Specification stage is more contextually specific than has been represented at the Statement stage, because specific contextual requirements have been assigned at this stage. Visual image is even more context-specific than the symbolic equation evidenced through the perceptual reality of visual image (O'Halloran, 2005, p. 111).

5.5.5 Knowledge progression: Coda where knowledge is summarized as a technical term

The final stage in this curriculum mini-genre is the Coda where the name of theorem is given. Pythagoras' Theorem was defined at the Statement stage and restated at the Specification stage. Contrasted with the above two stages, only a technical term, namely "*Pyth Theorem*", is rendered in the Coda. This abbreviated version is even more simplified than its original lexical items: "*Pythagoras'*

Theorem". The use of "technical terminology" (Halliday & Martin, 1993, p. 15) in construing knowledge is crucial in scientific discourses. According to Halliday, "technical terms are not simply fancy equivalents for ordinary words" (Halliday, 1993, p. 70). The emergence of a technical term is through recontextualising "the conceptual structures and reasoning processes" (Halliday, 1993, p. 70) into a nominal group. In mathematics, "the conceptual structures" (Halliday, 1993, p. 70) corresponds to the knowledge structure of the mathematical concept. The "reasoning processes" (Halliday, 1993, p. 70) correspond with the generic structuring through which the semiotic resources encapsulating the mathematical knowledge unfold progressively. Therefore, one technical term recontextualises both knowledge structure and generic structure. These two properties, which are possessive of very complex structures, have been encapsulated into a nominal group.

5.5.6 Recontextualisation between different components within a generic stage: focusing on Specification

The generic progression outlined in previous sections indicates how these different generic stages instantiated the knowledge of Pythagoras' Theorem differently. In this section, the focus has been shifted to argue how recontextualisation works within the same generic stage when it is composed of different components. The generic stage of Specification is the one where a mathematical equation coexists with the geometric image. The recontextualisation between these two components is analysed in this section. The discussion conducted in this section departs from the angle of the multi-semiotic nature of mathematical discourse, addressing how recontextualisation works between mathematical equation and geometrical diagram.

Both components within the Specification utilize multi-semiotic resources including mathematical symbols and a visual diagram. The theoretical foundation accounting for the multi-semiotic usages in mathematical discourse is that mathematical discourse is in essence multisemiotic. The construction of mathematical discourse normally involves semiotic resources in addition to langauge. In particular, mathematical symbolism and visual image are two predominantly utilized multi-semiotic resources in mathematical discourse

(O'Halloran, 2005) accompanying verbal language. In mathematical discourse, it is through recontextualisation that mathematical symbolism and visual image as the "new forms of semiosis in the mathematics" (O'Halloran, 2005, p. 26), replace classical and early ways of expressions. For example, mathematical symbols such as "+, -, ×, ÷" replace verbal expressions such as "plus, minus, multiple and divide" respectively, to generate a more economical way of writing mathematical language. This replacement, termed by O'Halloran as "semiotic adoption" (O'Halloran, 2007b, p. 92) indicates that "system choices from one semiotic resource are incorporated as system choices within another semiotic system" (O'Halloran, 2007b, p. 92). Therefore, there is a recontextualisation between linguistic elements and symbolic elements whereby "symbolic elements appear in linguistic statements" (O'Halloran, 2007b, p. 92), capable of functioning as the experiential categories such as participants, processes and circumstances. This functionality was originally demonstrated through verbal language. As for mathematics, this phenomenon where mathematical symbols function like verbal language, is because natural language is the metalanguage of mathematics. In the designing of mathematical theories, "the grammar of natural language" (Halliday, 2002, p. 392) is deployed and this grammar "enables mathematical expressions to be rendered in English, or Chinese, or other forms of distinctively human semiotic" (Halliday, 2003, p. 117). Following Halliday's (2002) elaboration, O'Halloran (2005) observes that "English is used as the metalanguage to teach mathematics" (p. 200).

Figure 5.47 is revisited here and sequentially re-numbered Figure 5.49. Its purpose is to suggest that at the clause level, the functionality of verbal language has been recontextualised into mathematical symbolism.

$$\text{In } \Delta ABC\text{, if } \angle C = 90°\text{, then } a^2 + b^2 = c^2$$

Figure 5.49: An example of Pythagoras' Theorem presented in mathematical symbolism (adopted from Wong & Wong, 2009, p. 103)

In this example, the mathematical symbolism recontextualised verbal language through providing an economical expression with less verbal language involved. Figure 5.50 is the linguistic expression when Figure 5.49 is rendered back into a non-economic manner of expression.

In triangle ABC, if corner C is equal to 90 degrees, then the square of leg a plus the square of leg b, is equal to the square of the hypotenuse c.

Figure 5.50: Symbolic expression is rendered back to linguistic expression

Evidently, Figure 5.49 is a more economical manner of expression, using less wording. This results from the recontextualisation between verbal language and mathematical symbolism where in this example, linguistic expressions have been taken placed by mathematical symbols. For example, "*triangle", "corner", "degree", "leg a", "square", "plus", "leg b", "is equal to"* and *"hypotenuse c*" have been recontextualised into symbolic forms such as *"Δ", "∠", "°", "a", "²", "+", "b", "="* and *"c*" respectively. The experiential meaning of the linguistic expressions, such as playing the important roles in acting like participants, processes and circumstances, have been replaced by the symbolic expressions. This type of recontextualisation, termed "Semiotic Adoption" by O'Halloran (2007b, p. 92) is now becoming an unmarked choice of written mathematics.

Semiotic Adoption could be further categorized into two subsections, glossed here as implicit recontextualisation and explicit recontextualisation. Still based on the examples in Figures 5.47 and 5.49, implicit recontextualisation indicates the instances where recontextualisation between linguistic expressions and symbolic expressions have "overcome the gravity well of specific context" (Maton, 2009, p. 55). For example, the relationships between "Δ" and "triangle", between "∠" and "corner", between "°" and "degree", between "²" and "square", between "+" and "plus", and between "=" and "is equal to", have been well established in contemporary mathematics. This type of recontextualisation, which is glossed as implicit recontextualisation, has now become the default expression and has overcome the gravity of a specific context. As for explicit recontextualisation, the recontextualisation between linguistic expressions and symbolic expressions is determined by the specific context. For example, the denotations between "a" and "leg a", between "b" and "leg b", and between "c" and "hypotenuse c" are only recognizable when the specific context is delineated; for example, the provision of a right-angled triangle. Otherwise, the recontextualisation between linguistic expressions and symbolic expressions is not bridged. This type of recontextualisation is glossed as explicit recontextualisation because an explicit requirement of a specific context is the pre-requisite to enable the recontextualisation.

The recontextualisation of the relationship between mathematical symbolism and verbal language outlined above, underpins how the symbolic equation in Figure 5.47 makes meaning. Mathematical symbolism within this symbolic equation has been divided into implicit recontextualisation and explicit recontextualisation. For mathematical symbols such as *"Δ", "∠", "°", "^{2}", "+", and "="*, their usages are no longer restricted to any specific context. For mathematical symbols such as "a", "b" and "c", their usage is still conditioned within a specific context. In this pedagogic item, the specific context underlined through the symbolic expressions such as "In ΔABC" and "if ∠C = 90°" has been visualized by the visual image. Recontextualisation between the symbolic equation and the visual image could be identified. Figure 5.48 is an example used by Wong and Wong (2009, p. 103) in association with Figure 5.47 when Pythagoras' Theorem is introduced in the textbook. This component is the visual diagram component.

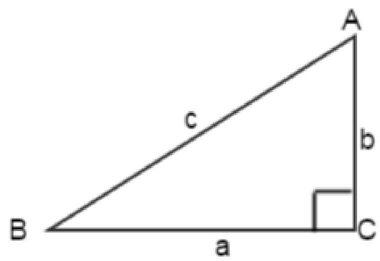


Figure 5.51: A geometric visual image (adapted from Wong & Wong, 2009, p. 103)

This geometric visual image is constructed from the following semiotic recourses: a geometric diagram as a triangle, with three capitalized letters, *A*, *B* and *C*; three lowercase letters, *a, b* and *c;* and one symbol, "┌". According to O'Halloran (2007b), the construction of this visual image is "semiotic mixing", which indicates that "linguistic and symbolic elements" synergize with "the visual display of geometric diagram" (O'Halloran, 2007b, p. 93). This synergy, as displayed in Figure 5.48, is that three capitalized letters, *A*, *B* and *C,* symbolise three corners; three lowercase letters, *a, b* and *c,* symbolise three lines; and the symbol "┌" marks that the corner is a right angle. These semiotic resources collaborate with each other to display a geometric visual image, which is used in association with the symbolic expression in Figure 5.47.

Figure 5.48 is a combination of both mathematical symbolism and a geometric diagram, displaying a contextual specific example of a right-angled triangle. This contextual specification lies in its explicit indication of the name for each edge, each angle and the angle that is the right angle.

Figure 5.47 is displayed alongside Figure 5.48 on page 103 in Wong and Wong (2009). Figure 5.50 displayed how these two Figures are combined, with dotted lines indicating the indexical relationship between elements in the symbolic equation and elements in the visual image.

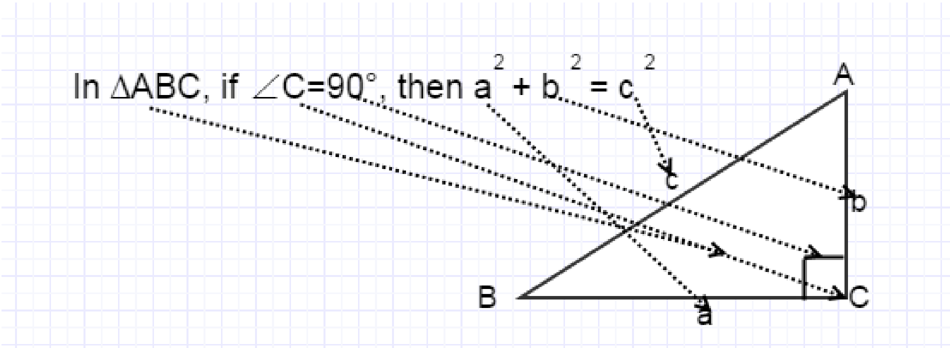


Figure 5.52: Identification relationship between Figure 5.47 and Figure 5.48

Table 5.13 exemplifies the indexical co-relation between the elements in the symbolic equation and the elements in the visual image.

Table 5.13: A corresponding "indexical" relationship

Type of relationship	Symbolic elements in Symbolic equation	Visualized elements in Visual image
indexical relationship	ΔABC	A triangle ABC
	∠C	Corner C
	90°	┌
	a	Edge a
	b	Edge b
	c	Edge c

In Table 5.13, there is a corresponding "indexical" (Martin & Rose, 2008, p. 169) relationship between elements in the symbolic expressions as in Figure 5.47 and elements in the geometric expressions as in Figure 5.48. For example, "ΔABC" is indicated by "three corners each marked with a capitalized letter", "∠C = 90°" is indicated by "a combination of symbol '┌', the corner and the capitalized letter C", and edges which are symbols in Figure 5.47 indicated by lowercase letters (a, b and c) are converted into a combination of both symbol and the immediate neighbouring edge.

Experientially, the circumstances in Figure 5.47 are "In ΔABC" for the scope, and "if ∠C = 90°" for the condition. These two types of circumstances determine the premise under which different participants could coordinate with each other.

The visual image in Figure 5.48 visualizes these two circumstances with the help of both symbols and a geometric diagram. The visual image helps to contextualize Figure 5.47 by explicating the circumstances.

Therefore, the recontextualisation between symbolic expressions in Figure 5.47 and visual image in Figure 5.48, is that Figure 5.48 recontextualises Figure 5.47 partially, instantiating and visualizing the circumstances with the help of its discernible visual and symbolic features. As the recontextualisation is partial, it indicates that the other two crucial parts of experiential meaning, the participants and the process, have not been recontextualised into the visual image. In composing the generic stage of Specification, visual imagery plays an auxiliary role in presenting the knowledge of Pythagoras' Theorem. Inter-textually, its major function is to recontextualise the circumstances in the symbolic equation.

5.5.7 Ideational commitment in each component: some affiliated functions of mathematical visual images and symbolic equations: cues to other pedagogic items

Previous subsections considered the recontextualisation from the viewpoint of two parameters: a vertical parameter is assigned to understand the progression between different generic stages and a horizontal parameter is assigned to understand recontextualisation between different components within a generic stage. In this section, the approach of ideational meaning commitment is applied to illustrate how each component within this pedagogic item has contributed to commit the experiential categories of participants, processes, and circumstances in compliance with the recontextualisation perspective outlined before. Figures 5.53, 5.54 and 5.55, are three illustrations dedicated to the elaboration of the experiential categories within three different components.

Statement: In a right-angled triangle, the sum of the squares of the two legs is equal to the square of the hypotenuse.		
Participants:	the sum of the squares of the two legs	the square of the hypotenuse.
Process:	is equal to	
Circumstance:	In a right-angled triangle,	

Figure 5.53: List of experiential categories identified in statement

Symbolic equations: In ΔABC, if ∠C = 90°, then $a^2 + b^2 = c^2$			
clause structure:	Hypotactic Structure in "regressive sequence" (Halliday & Matthiessen, 2004, p. 393).		
	a conditional dependent clause:	the major clause: *then $a^2 + b^2 = c^2$*	
	if ⍰ C = 90°,		
Participants:	⍰ *C* and *90°*	a^2, b^2, and c^2	
	relational process: =	*operative process: +*	relational process *: =*
Circumstance	"In ΔABC" specifies the range		
	"if ∠C = 90°" specifies the condition		

Figure 5.54: List of experiential categories identified in symbolic equations

Participants:	edges: a, b, c
	points: A, B and C
	corners: ∠A, ∠B and ∠C
	∠C is a right angle. This property has been indicated through the use of symbol " ┌"
Process:	Static process of existence
Circumstance	A triangle

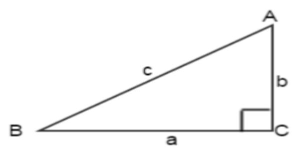


Figure 5.55: List of experiential categories identified in visual images

The three figures presented above outline the experiential categories of participants, processes and circumstances within the statement, the symbolic equation and the visual image in the pedagogic item.

Recontextualisation between different components has been investigated in previous sections by discussing the co-related nature revealed by the recontextualised relationships. However, when we investigate recontextualisation, we should not overlook the experiential potential that different components commit. Following a linguistic approach of reinstantiation, each component has its own distinctive way of instantiating Pythagoras' Theorem. Since this study has reconciled recontextualisation with reinstantiation to investigate how Pythagoras' Theorem, as the baton, has been relayed between different components within a pedagogic item, ideational meaning commitment has been utilised as the analytical model to account for the distinction and connection between different components.

There is always a tidy complementarity lacking between different components; rather each individual component has the potential to commit the experiential categories that are missing in others (Painter et al., 2013). These isolated experiential features are the extension of others in terms of the range of involvement.

Regarding the commitments of participants identified in different components, the correspondence between participants in different components is not achieved through a one to one co-relation. Figure 5.56 enumerates the participants identified in each component.

Experiential Category	**Statement**	**Symbolic Equation**	**Visual image**
participants:	the sum of the squares of the two legs; the square of the hypotenuse.	$\angle C$, $90°$, a^2, b^2, c^2	edges: a, b, c points: A, B and C corners: ∠A, ∠B and ∠C

Figure 5.56: Identification of Participants in three components

Through an enumeration of participants, there is the lack of a "one to one" relationship. Rather, a hierarchical cline has been assigned to inter-relate participants in different components. Figure 5.57 provides a hierarchical cline between participants in different components.

Participants in the Statement	the sum of the squares of the two legs			the square of the hypotenuse.
Rank-shifted in the symbolic equation	$A^2 + b^2$: A rank-shifted structure has been identified between the participant in the Statement and its symbolic equivalence in the symbolic equation. The nominal group in the Statement has been converted into a clause indicted by the operative process "+"			
Participants in the symbolic equation	a^2 and b^2	Processes in the symbolic equation	+	c^2
Rendering "^{2}" into operative process	Both a^2 and b^2 are participants in the symbolic equation, converted from a × a and b × b through the use of spatial notation ^{2}. If the special notation is unpacked, the nominal groups in the symbolic equation will be converted into two clauses: a × a and b × b, with participants linked by the operative process "×".			c^2 is a participant in the symbolic equation, converted from c × c through the use of spatial notation ^{2}. If the special notation is unpacked, the nominal group in the symbolic equation will be converted into one clause: c × c participants linked by the operative process "×".
Participants in the visual image	Edge a and Edge b are the visual participants in the visual image. These two edges are rank-shifted from a^2 and b^2 Operative process "×" is omitted in the visual image			Edge C is the visual participant in the visual image. This edge is rank-shifted from c^2. Operative process "×" is omitted in the visual image

Figure 5.57: A hierarchy of rank between participants in different components

Based on Figure 5.57, a hierarchy of rank between different participants could be identified with the nominal group in the statement being composed of a nominal group plus a process in the symbolic form. Following the categorisation by Hood (2008) of different mechanisms of ideational commitment, "Infusion" as the means could be mapped onto the hierarchical cline between different participants. The nominal group in symbolic form is infused with a process. This process could not be reflected in the geometric participants in the visual image. The use of rank-shifted participants signals the property of the Theorem as a pedagogic item, as a mini-genre. Of course, it is easily observable that the participants committed by different components are different; however, it is more important to identify a structure of rank shifting of participants following a cline of hierarchy in the movement between the Statement, the symbolic equation and the visual image.

Regarding the different commitments of processes, relational process has been committed in the Statement in the linguistic expression of "is equal to". This linguistic expression has been encoded as a symbolic form of "=", rendering the same experiential meaning of its linguistic expression. As has been noted before, operative processes such as "+", in the symbolic equations are infused with the spatially notated participants such as "a^2" and "b^2" to stand for participants in the Statement. Therefore, when the linguistic expressions are symbolically expressed, more processes will occur resulting from the linguistically nominalised forms. With reference to visual images, the major processes, "spatial, temporal and relational with entities in the form of the line segments, circles and curves requires (sic) explanation" (O'Halloran, 2005, pp. 43–44). This relational process is omitted in the diagram indicating "a loss of meaning" (O'Halloran, 2005, p. 134) in the dialogue between the statement and the visual image.

In this example, as has been justified before, the knowledge structures about Pythagoras' Theorem across the three stages, namely Statement, Specification and Coda, remain the same, enabling the discussion in this section to be manageable. Later, in the succeeding analysis and discussion chapter, the complexity in terms of the knowledge structure of Pythagoras' Theorem in its own right will emerge. Focusing only on the generic development of information in this pedagogic item, the generic structure progresses in the form, Statement ^ Specification ^ Coda. The technical term appearing at the Coda stage, recontextualises what was presented in the previous sections. Experientially, the technical term recontextualises participants, process and circumstances that have been identified in the preceding two stages and summarises them all in one nominal group. This nominal group is an encapsulation of everything that has been mentioned, applicable to all the circumstances, either a mathematical-specific context such as was presented in the Statement stage or an example-specific context such as was presented in the Specification stage.

5.5.8 Rendering an analytical model within one pedagogic item: knowledge delocation and relocation

The previous sections rendered an analytical model that is workable in understanding the delocation and relocation of knowledge within one curriculum mini-genre. This model is to consider both “the conceptual structures and reasoning processes” (Halliday, 1993, p. 70) for each generic stage within one curriculum mini-genre. The conceptual structure is related to the knowledge structure while the reasoning process is associated with the generic structuring indicated through the lexico-grammatical relationship. In the discussions, the relationship was concerned with the nuclear relationship between lexical items within the clause. The concern was also with how these nuclear relationships were bridged in terms of the contextual dependency level with which each generic stage was tied.

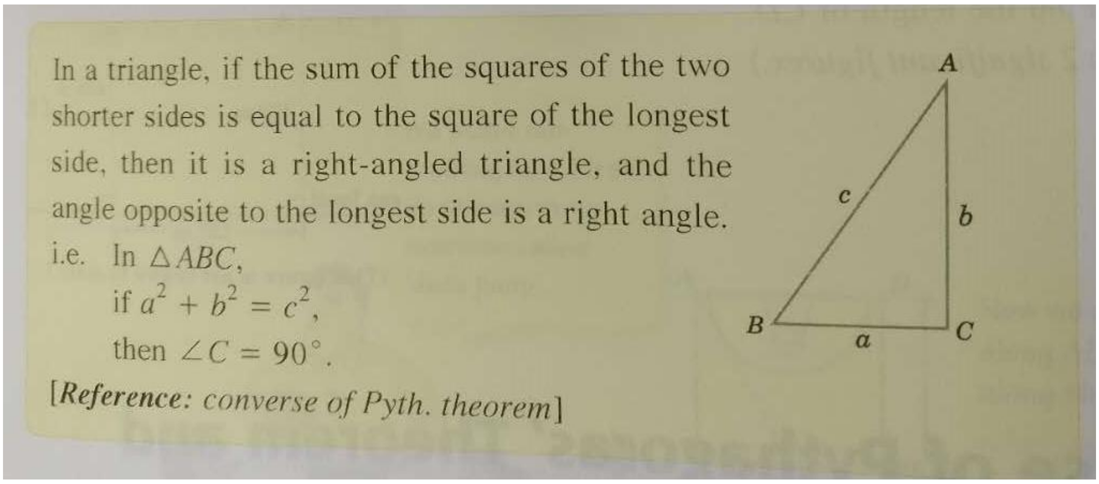
Following the direction of how the generic structure develops, i.e. Statement ^ Specification ^ Coda, Pythagoras’ Theorem as the central mathematical knowledge, progresses in this generic structure. It was first instantiated in the statement in linguistic expressions, and tied to the mathematical contexts marked with the nominal group: “in a right-angled triangle”. Pythagoras’ Theorem was then delocated from the Statement and relocated into the next stage: Specification. In this stage, the semiotic resources used here have shifted into a symbolic equation accompanying a geometric diagram, which visualizes the circumstances under which the symbolic equation works. Compared to the representation of Pythagoras’ Theorem in the Statement, the representation at this step is bonded with a specific context, namely “In ΔABC” and “if ∠C = 90°”. That means as mathematical knowledge, Pythagoras’ Theorem at this stage is dependent on its specific context. At the final stage within this curriculum mini-genre, Pythagoras’ Theorem was relocated from the symbolic equation and was delocated into a technical term. The contextual dependence level of this form of representation is the same as that in the Statement. Pythagoras' Theorem is workable once the context has been specified as a right-angled triangle. The semiotic resources used in the technical term are simplified into a nominal group with two words forming the Classifier + Thing structure.

This delocation and relocation of knowledge, moving from contexts specified in mathematics to the context specified in the particular example, and then later returning to the mathematical-specific contexts, are recurrent methods of introducing knowledge in the genre termed "Theorem". The recurrent configurations are identifiable across different examples when new theorems are introduced in mathematics.

5.5.9 The generic structure of Theorem as a type of genre

The generic structure in the form of "Statement ^ Specification ^ Coda" identified in Wong and Wong (2009) is recurrently found in other recommended mathematical textbooks. Figures 4.4 to 4.11 display these instances, showing the representation of Pythagoras' Theorem in other mathematical textbooks. These instances represent complimentary research data, justifying the meaning-making process generalised from Figure 4.3, could be applied in other similar mathematical texts. Halliday (2004a) identified that "writers of scientific textbooks often recapitulate the process as a way of introducing technical terms to the learners" (2004a, p. 88). In this study, this phenomenon could be extended to account for the recapitulation of the same process adopted to introduce technical terms in different mathematical textbooks based on the comparison between those textbooks, as seen in Figures 4.3 to 4.11. This is so, since the exploration of how Pythagoras' Theorem as a technical term is introduced in this instance, could be accounted for in the introduction of other mathematical concepts in this textbook (Wong & Wong, 2009) and in others (Figures 4.4 to 4.11).

A similar generic structure could also be identified in introducing other mathematical concepts. For example, Figure 5.58 displayed how *Converse of Pythagoras Theorem* has been introduced in Wong & Wong (2009). Figure 5.59 displayed the annotated version of this mathematical concept.


In a triangle, if the sum of the squares of the two shorter sides is equal to the square of the longest side, then it is a right-angled triangle, and the angle opposite to the longest side is a right angle.

i.e. In $\triangle ABC$,

if $a^2 + b^2 = c^2$,

then $\angle C = 90°$.

[*Reference: converse of Pyth. theorem*]

Figure 5.58: The Introduction of Converse of Pythagoras' Theorem (Wong & Wong, 2009, p. 120)

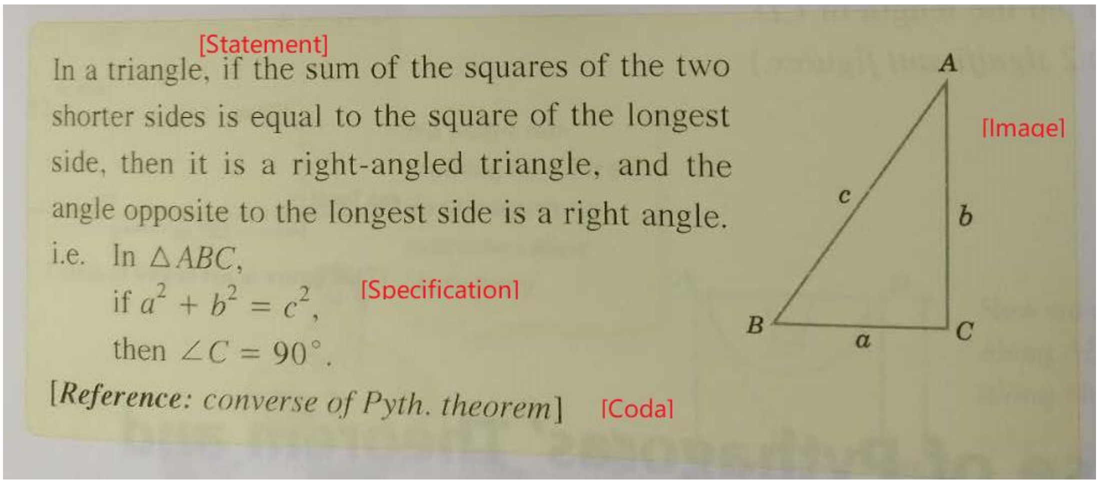


Figure 5.59: Annotated version of Figure 5.58

As could be seen in Figure 5.59, its generic structuring is Statement ^ Specification ^ Coda with one geometric image positioned alongside the verbal statement, symbolic equations and technical term. With the help of this example, we can see that the generic structure identified for Pythagoras' Theorem is not isolated. This structure has been adopted for a wider scope when mathematical concepts are introduced as pedagogic items in textbooks.

5.6 Summary of this chapter

This chapter discusses the different instantiations of the same mathematical knowledge – Pythagoras' Theorem – regarding pedagogic discourses outside classroom settings taken from the partial curriculum ecology. The instances in the Syllabus (EDB, 1999) progress accumulatively to establish the knowledge structure of Pythagoras' Theorem by providing both its internal characteristics and illustrative figures such as conceptual tabulations and flowcharts. Semiotic resources adopted in the Syllabus (EDB, 1999) are provided through a combination of verbal language, symbolism and visual images, with the relationship established between different instances being the lexical repetition of the linguistic resources of "Pythagoras' Theorem". It is noteworthy that it is in the Syllabus (EDB, 1999) that the taxonomic relationship regarding Pythagoras' Theorem has been reconciled with the conceptual network structure in the knowledge construction stream, preparing for the analysis in later sections. When the discussion moves to the curriculum guideline (HKEAA, 2007), the knowledge structure regarding Pythagoras' Theorem has been embedded, not in the diagrammatic representation, but in the taxonomic and logical relationship at both linguistic and symbolic level identified in Text Box 5.1. Although presented as a lexical item, the knowledge structure regarding Pythagoras' Theorem should be inferred based on the taxonomic relationship and logical relationship through careful analysis at semiotic level. Pythagoras' Theorem is a requirement in the curriculum guideline, and its knowledge as well as its semiotic resources, is a pre-requisite to serve as the prompt in understanding the curriculum guideline. An even more demanding pedagogic item is that of the assessment (HKEAA, 2012). This pedagogic item is an instance of instantiation of Pythagoras' Theorem without mentioning the lexical term "Pythagoras' Theorem". Students need to infer that term based on the available semiotic resources. The analysis of the assessment (HKEAA, 2012) identifies the fundamental prompts from which, without prior knowledge of mathematics, students could still identify the ideational meaning absent from each component, such that they could proceed to noticing the missing ideational meaning. To solve the assessment task, students still need to possess the knowledge of Pythagoras' Theorem. The totality of the mathematical knowledge of Pythagoras' Theorem lies in the analysis of the pedagogic item of the Theorem. The analysis of the Theorem comprehensively reveals how the totality of one mathematical concept could be demonstrated by focusing on its semiotic construction and the generic development. To solve the unsettled issues in the assessment tasks in HKEAA

(2012), the totality of the one considered in the textbook (Wong & Wong, 2007, p. 103) could be delocated from its original setting and relocated amongst other pedagogic items.

The delocation and relocation of mathematical knowledge such as Pythagoras' Theorem is the gateway that, we as linguists could prepare for our students, in understanding the different pedagogic discourses given the central focus has been placed on the understanding of the recontextualisation relationship within and between different pedagogic items. The understanding of the semiotic construction of pedagogic items is as important as the understanding of mathematical knowledge. In secondary school years, students have been told that the more one practices, the better the score is achieved. Actually, to practice more does not naturally equate with familiarity with the recontextualisation between knowledge and the semiotic resources. The solution to the assessment tasks are implicit and need to be determined by the students themselves. Here, in this section, a social semiotic approach aims to bridge the gap between knowledge and semiotic resources, empowering the students with the literacy skills in uncovering the knowledge hidden within the semiotic resources, and relate that knowledge with existing knowledge and existing semiotic resources.

To understand the knowledge is a twofold issue. It comprises the need to understand the complex structure of knowledge and to understand the semiotic resources used to realise the knowledge. Therefore, a scale of semiotic combinations, or a range of semiotic resources, is required so that the Semiotic Cohesion (O'Halloran, 2007b) could be identified with different semiotic resources being bridged. The reason why the assessments could not be solved is chiefly because we are constrained to the semiotic resources at hand, and could barely de-contextualise the semiotic resources (Coffin & Donohue, 2014) previously used for underlining, explaining and describing the mathematical knowledge from its original context, thus bridging these semiotic resource with the troublesome semiotic resources used in the assessment.

Of course, it is highly demanding to comprehend every mathematical concept, to comprehend its complicated internal characteristics, and to comprehend its relationship with other mathematical concepts. The conceptual requirements are the tasks to be accomplished by all the education stakeholders implied by the curriculum ecology. This research could provide valid ways to describe the recontextualisation required in the mathematics curriculum, with an orientation toward the knowledge that has been embedded (in semiotic resources), delocated (from the original semiotic situation) and relocated (into other semiotic situations). Through the process of recontextualisation, different instances of the same knowledge are bridged. Without explicating the required knowledge, the cohesive relationship between the semiotic resources used for introducing the required knowledge in pedagogic discourses, such as textbooks, and the semiotic resources used for assessment in pedagogic discourse, such as the examination papers, could be bridged. Their explicit connection is through the external semiotic resources. This explicit connection could inform their implicit knowledge connection.

Chapter Six – Findings and discussions

Chapter Five discussed how Pythagoras' Theorem has been represented differently in the curriculum ecology. Differences lie in the purposes and functions of different pedagogic discourses. The analysis has reconciled recontextualisation with reinstantiation, attempting to argue the sociological understanding of knowledge representation from the social semiotic approach, empowered with linguistic evidence.

Drawing from the analysis rendered in Chapter Five, this chapter discusses the findings that this study has identified. In Section 6.1, I argue how knowledge structure could be underlined with the help of taxonomic relations that emerged at the linguistic level. In Section 6.2, I discuss multi-semiotic issues that emerged from the analysis, confirming the meaning-making processes in mathematics as a multi-semiotic process. In Section 6.3, I discuss recontextualisation . In Section 6.4, I extend recontextualization and outline decontextualisation of knowledge. In Section 6.5, I address issues on Commitment and knowledge representation in mathematics. In Section 6.6, I discuss the nature of mathematics. In Section 6.7, I further elaborate the nature of mathematical cues. In Section 6.8, I elaborate the inter-semiotic nature of mathematics. In Section 6.9, I suggest the contribution that this study can makes to informing education stakeholders with an analysis of another mathematical concept to emphasize the appliable nature of this study. In Section 6.10, I offer a linguistic explanation to the "Needham Grand Question" through investigating the name right for Pythagoras' Theorem.

Based on these findings and discussions, this study has contributed to associate the Ideation system from the discourse semantic approach (Martin & Rose, 2003, 2007, 2014) with the sociological approach to knowledge structure suggested by Bernstein (2000). With the help of the Ideation system, both the horizontal knowledge structure and the hierarchical knowledge structure have been subcategorized. This study has also reconciled recontextualisation with reinstantiation and theorized recontextualisation from the social semiotic perspective. Recontextualisation, which is a sociological model, proposed to investigate knowledge delocation and relocation has been imbued with linguistic methods.

In the course of presenting the findings identified in this study, evidence based in the analysis from Chapter Five, will be cited as necessary.

6.1 Knowledge structure, taxonomic relations and expected knowledge

One of the major efforts that this study has focused on is to investigate the knowledge structure of mathematics from an SFL-informed approach.

In the Syllabus (EDB, 1999), Pythagoras's Theorem as well as other mathematical concepts is introduced predominantly in the form of a textual exposition. This way of introducing mathematical concepts is not isolated. Indeed, nearly every mathematical concept in the Syllabus (EDB, 1999) is considered using textual means. Taking Pythagoras' Theorem as an example, in Tables 5.1, 5.2 and 5.6, textual layouts in the form of a tabular taxonomy and flowchart format offers a straightforward way of delineating the relationship between Pythagoras' Theorem and other mathematical concepts. Both the tabular format and flowchart are regarded as a "conceptual representation" (Kress & van Leeuwen, 2006, Chapter 3) in terms of the relationship at lexical level with respect to their textual resources, such as the usage of juxtaposition and alignment, the compositional arrangement and the sequence of the information unfolding.

The conceptual relationship between different mathematical concepts indicated through textual resources could be converted to the knowledge structures following Bernstein's (1999) categorization. The compositional relationship encoded in the tabular taxonomy and the sequential order encoded in flowchart, are both informing the hierarchical knowledge structure (Bernstein, 1999).

As for the curriculum guideline (HKEAA, 2007), lexical items concerned with the instantiation of Pythagoras' Theorem have been considered from the perspective of how their lexical relationship is established. Unlike the Syllabus (EDB, 1999) where the textual layout using a tabular taxonomy and flowchart format, underlines the relationship between different lexical items, the establishment of a lexical relationship in the curriculum guideline (HKEAA, 2007) is achieved through both textual inference and ideational inference. The taxonomic relationship that is one of the theoretical systems within the Ideation system (Martina & Rose, 2003, 2007, 2014), has been found to be the major impetus behind the textual inference of lexical strings between different lexical items. As for ideational inference, contextual features such as the compositional relationship, causality and sequencing are found to be the catalysts [6] underpinning the lexical relationship.

One major finding emerging from the curriculum guideline is that lexical strings informed by the taxonomic relations could be converted to account for the relationship at a conceptual level, indicating the horizontal knowledge structure or hierarchical knowledge that exists between different lexical items. For example, in Chapter Five, the lexical strings concerned with "one of the properties of vectors" have been converted into different types of knowledge structures indicated by the different types of taxonomic relationships between lexical items related to "one of the properties of vectors" (See Section 5.3.1 to Section 5.3.5).

To be more specific, based on the analysis in Chapter Five, the lexical relations such as synonym and contrast (Martin & Rose, 2003, 2007, 2014), correspond to the horizontal knowledge structure at the knowledge construction level. For instance, Figure 5.20 displayed the converse relationship between *"Pythagoras' Theorem"*, and *"one of the vectors to be a zero vector"*. This converse relationship has been converted into the horizontal knowledge structure since these two mathematical concepts together form "the geometrical meaning of the property of the scalar product of the vectors". As for the lexical relations such as part-whole relationship and class-subclass relationship, they could be converted into a hierarchical knowledge structure, offering delicate subcategories of a hierarchical knowledge structure. For instance, based on the analysis in Chapter Five, "Pythagoras' Theorem" is a kind of "geometrical meaning of the property" while "two perpendicular vectors a and b with the same initial point" is a part of "Pythagoras' Theorem".

[6] A chemical metaphor is used here.

Following Martin and Rose (2003, 2007, 2014), a taxonomic relationship is a system in underlining the relationship between different lexical items. For example, their terms, “cold” and “hot” are antonyms because the conceptual relationship between “cold” and “hot” has been established with regard to converse roles in their physical and natural states. As for the technical terms of mathematics that this study is concerned with, what is the relationship between different mathematical concepts requires further exploration. It is assumed by EDB (1999) and HKEAA (2007) that students and teachers are following the pre-decided by EDB (1999), conceptual relationship between different mathematical concepts. For example, the pre-decision between different technical terms of mathematics at conceptual level is related to the “expectancy” of the connection between different concepts (Martin & Rose, 2014, p. 117). This “expectancy” enables the translation between knowledge structure at a conceptual level and lexical relations at a linguistic level. For example, when I claim that “Pythagoras’ Theorem” is a kind of “geometrical meaning of the property” at the lexical level, it is because “Pythagoras’ Theorem” has been pre-decided by the EDB and the HKEAA to be hierarchically organised with “the geometrical meaning of the property” at the conceptual knowledge level. Therefore, the expected knowledge structure will inform the lexical relations through the explication of the potential lexical strings between different lexical items.

An obvious paradox emerging from the pre-decided expectancy of knowledge structure is whether the readers are, or are not, familiar with the expectancy of the knowledge structure. How could we as linguists still guide the readers to understand the knowledge structure embedded within the semiotic resources at hand? This is a major concern of the present study. For example, we could provide guidance to teachers and students, helping them understand the compositional relationship, causality and sequencing between different lexical items in mathematics and other subject.

This paradox exists in the curriculum guideline when verbal text has been adopted as the major semiotic resource in presenting the lexical relationship between Pythagoras’ Theorem and other mathematical concepts. In the course of the verbal text, the knowledge structure between different mathematical concepts is still pending confirmation. In order to infer the knowledge structure, verbal text is the only semiotic resource readers could rely on. Based on this premise, Causality has been prepared as the model in underlining the hierarchical knowledge structure. For example, Figure 5.26 explained how the cause-effect relationship between *Pythagoras' Theorem*, *the hypotenuse is |a-b|* and *a•b = 0* is established with the help of causal conjunction "hence", overcoming the deficiency of textual evidence at word group level. As has been found in Figure 5.26, in terms of the horizontal relationship, paratactic relationship between *"one of the properties of the scalar product of vectors"* and *"*$|a-b|^2 = |a|^2 + |b|^2 - 2\,(a \bullet b)$*"* informs the horizontal relationship.

Based on the curriculum guideline, evidence applied to infer the knowledge structure has been explored, not at the nominal group level, but with the help of the clause structure. A model displays the mutual informing relationship between knowledge structure and linguistic resources is offered. This mutual informing relationship could be figuratively portrayed as two parallel lines: one for knowledge structure and the other for linguistic relationship. Figure 6.1 displays this process of mutual informing.

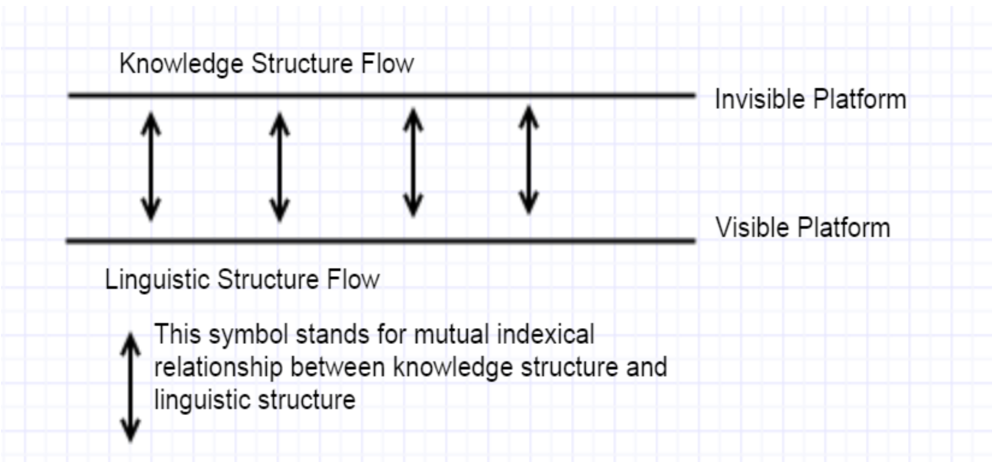


Figure 6.1: Mutual indexical relationship between knowledge structure and linguistic resources

As can be seen in Figure 6.1, two parallel lines, one for knowledge structure and the other for linguistic relationship, have been visualised. Knowledge structure exists at the invisible platform representing conceptual concepts, while the linguistic relationship exists at the visible platform organised by linguistic resources. The relationship between knowledge flow and linguistic structure flow has been formulated as a symbiotic relationship. These two platforms mutually inform each other (symbolised as the bidirectional arrow "↕").

Based on that model, the tightly intersected relationship between knowledge structure and linguistic resources suggests the symbiotic relationship between them in the process of both construction of knowledge and the composition of linguistic resources at discourse level.

The symbiotic relationship between knowledge structure and linguistic resources drives the progression of knowledge and the complication of semiotic resources adopted in the visualization of knowledge. The complication of semiotic resources starts from the involvement of linguistic resources at the morphological level such as the use of suffixes in the classification of certain types of lexical families. For example, in Martin (2013), the suffix "-ism" as the morphological resource within the lexical items has been utilized as the means by which to label different ideological streams in the history of philosophy. This is effected by classifying their taxonomic relations as serial lexical items (such as modernism and post-modernism) or opposite lexical items (such as Communism and Anti-Communism) at the lexical relationship level. The morphological resources indicating the lexical relationship of Contrast will imply that different lexical items sharing the same suffix such as -ism are organized as the horizontal knowledge structure.

Expanding from morphological resources to linguistic resources, the semiotic resources adopted to construct knowledge become complicated. In order to interpret the lexical items that share no clue of the same suffixes, the interpretation of the lexical relationship relies heavily on the internal and invisible knowledge structure between these lexical items. Therefore, from a purely linguistic approach, the taxonomic relationship between different lexical items is determined by the conceptual relationship between them.

However, how could we underline the expected knowledge and expected knowledge structures with no sign of suffixes at lexical level? Nuclear relationship and activity sequences are two proposed systems. The nuclear relationship underlines how linguistic resources could predict the knowledge structure between different lexical items at the sentence level. The Activity sequence suggests how the embedded relationships between different events are presented as different sets of co-related activities. These two systems are useful methods both to understand the knowledge structure and to establish the expected knowledge and knowledge structures.

As for the nuclear relationship, from Figures 5.42 to 5.45, I demonstrated the nuclear relationship in the statement "In a right-angled triangle, the sum of the squares of the two legs is equal to the square of the hypotenuse". Without any prior knowledge of what Pythagoras' Theorem is about, the nominal groups (right-angled triangle, sum, leg, square, hypotenuse) together with the relational process (is equal to) are the prerequisite knowledge that Pythagoras' Theorem intergrates.

As for activity sequence, the temporal relationships identified in Figure 5.11 suggest that *the concepts of magnitudes of vectors* and *the scalar product of vectors* are prerequisite knowledge of *the geometrical meaning of the property* with the assistance of the temporal preposition phrase: *in previous lessons.*

The way in which the knowledge structure is presented in the curriculum guideline (HKEAA, 2007) is very different from the Syllabus (EDB, 1999). Knowledge structures between different mathematical concepts have not been transparently provided. Analysis of the curriculum guideline (HKEAA, 2007) suggests that if the relationship between different lexical items is implicitly mentioned at the lexical level, knowledge structures could still be effected with the help of a nuclear relationship and activity sequence. This is because the invisible knowledge structure could be uncovered when sufficient support is provided for the nuclear relationship and activity sequence.

6.2 Some multi-modal/multi-semiotic issues

Section 6.1 discussed the mutually informing relationship between linguistic resources and knowledge structure when Pythagoras' Theorem is instantiated in the Syllabus (EDB, 1999) and curriculum guideline (HKEAA, 2007). Verbal language and tabular layouts are the major semiotic resources applied in these two pedagogic discourses. This section discusses the multi-modal/multi-semiotic issues that this study has considered. Drawing from the analysis of the textbook and examination paper, the central issues concerned with multi-modal/multi-semiotic studies arise from the discussion of the relationship between different modalities. Taking the heated discussion on bi-modal discourse as an example, the predominant arguments are concerned with the underlying relationship between verbal language and visual imagery. The relationship between visual text and verbal text is concerned with elaborating, extending or enhancing, drawing. These types of relationship are drawn from Halliday's (1994) investigation of the relationship between different clauses within a clause complex. Significant work in this area includes the studies by Royce (1998, 2002, & 2007) and Martinec and Salway (2005). These works transformed the relationship between different clauses directly to account for the relationship between visual images and verbal resources. This transformation not only overlooks the very intimate dialogical relationship between these two modalities but also overlooks the meaning-making potential pertaining to each modality. Visual-verbal synergy extends far beyond the clause-complex relationship. In most cases, bi-modal texts do not appear to present a tidy complementarity (Painter et al., 2013) between different modalities. Therefore, in order to reveal how the experiential meaning has been committed by each modality, "committing" (Painter et al., 2013) which derives from the parameter of instantiation, has been adopted as the analytical model in this study.

As could be seen in the analysis of the textbook and examination paper, the relationship between geometrical visual image and the linguistic resources is that each modality commits the experiential meaning that is missing in others. They are mutually dependent in such a way that each modality commits a certain degree of meaning that the other modality does not. This notion could be extended to include tri-modal texts, covering a variety of multi-modal texts under various contexts. Speaking of the research data analysed in this study, tri-modal text has been specifically targeted as the typical mathematical text where linguistic description in the form of verbal language, mathematical equations in the form of mathematical symbols and geometrical diagrams in the form of visual images, co-exist to account for the composition of the mathematical text. Modality as the terminology used by Painter and her colleagues (Painter et al., 2013) to label both verbal text and visual image has been converted as a component. Component, which is a term invented by Cheong (2004), has been used to account for spatially aligned, juxtaposed and separated elements in one printable page-like document. The major components within a mathematical text are verbal text, mathematical equation, and visual image. In a mathematical text, each component is an individual element and has been spatially aligned with, juxtaposed with and separated from, other components.

The position taken in this study is inherited from this theoretical viewpoint that different modalities identified in the selected pedagogic item consist of several independent components. Each component is itself an instance of knowledge representation. They together compose the totality of knowledge representation with each component committing a certain degree of experiential meaning which is associated with the knowledge (in this study, the knowledge is Pythagoras' Theorem). Followed by the comparison between these different components, the interplay between different components has been modelled from an experiential meaning commitment perspective. In what follows, a fundamental step taken in this study is not to pre-designate the co-relation between different modalities but to treat each component as an independent element. By utilizing Halliday's work on clause complex, some existing research (Royce, 1998, 2002, 2007; Martinec & Salway, 2005) has a tendency to proclaim that visual image and verbal text are naturally co-related in the manner that visual image is dependent on verbal texts.

This study has unravelled the experiential meaning commitment for each component in the very first place. Departing from this point, this study co-related the different experiential meanings with reference to how these components are bridged. As has been found in the analysis, the relationship between different components are not in a tidy complementarity, confirming what Painter and her colleagues (Painter et al., 2013) found. The elements, which could not be assigned in a correspondent relationship, have opened up an unlimited meaning potential, ensuring them to be functional when other components are omitted, replaced or changed. For example, the visual image in Figure 5.4.1 (the theorem) has committed the participants of corners, the static process and the circumstance of location that have all been omitted in the verbal text and the symbolic equation.

This theoretical position could be further applied to deal with the situations where one component (such as the visual image) has been reproduced for a number of different contexts accompanied by different components. In each condition, the total ideational meaning committed by the visual image is preserved, but when it was bridged with different components, more or less meaning, or different meanings, might arise through the dialogical relationship. For example, once the visual image in Figure 5.4.1 has been utilized in a situation, other than the one discussed in this study, the experiential meaning that arises to produce the dialogical meaning is highly dependent on the context in which this visual image is placed.

The next section will discuss how recontextualisation has helped to make the meaning-making process in mathematics, achieve progress.

6.3 Recontextualisation: enriched with linguistic tools

In this present study, recontextualisation has been positioned as a research model reconciled with reinstantiation. This reconciliation of these two theoretical considerations is the contribution that I make to the field of knowledge construction and curriculum design.

Recontextualisation is a sociological interpretation of ontological and logo-genetic exchanges in discourse studies. Ontological recontextualisation addresses the exchanges over time. For example, in a recently completed PhD thesis within the Department of English, The Hong Kong Polytechnic University, Nancy Guo (Guo, 2015) has investigated how Chinese secondary school English textbooks have developed over the nine-year compulsory education system, highlighting that text types have changed over time, moving from more dialogical text types to more argumentation-oriented text types. Speaking from a sociological perspective, the development over time reflects the ontological parameter of information progression in which the same type of pedagogic discourse (in her work, the textbooks) has been investigated from the perspective of how they developed from the first year (primary year one) to the terminal year (senior secondary year three). This perspective is informed by Christie's (2012) work where she demonstrates how the school system for both history and science has developed from primary school to secondary school. Apart from the work concerning the school system development, the ontological perspective has also been applied to address how children's oral language develops (such as the work by Hasan 2005, Painter 1999) and how children's writing skills become progressively mature in school discourse (Christie & Derewianka, 2007). Noticeably, a variety of studies have investigated the ontological development of language, enriching the understanding of how recontextualisation works ontologically.

However, few studies have contributed to the understanding of how recontextualisation works from a logo-genetic perspective, looking at how one discourse reshapes others in the field of education. The point of departure for this research is to understand how mathematical knowledge has been de-contextualised from one discourse and is recontextualised into another discourse. This process is termed recontextualisation, consisting of the continuing motion of de-contextualisation and recontextualisation. Knowledge, which has been argued in this study as the "thing" being recontextualised, was discussed in Chapter Two in which the underlying property of mathematical knowledge is addressed. To be more explicit, in this study, mathematical knowledge is invisible and the analytical unit for mathematical knowledge is one mathematical concept. One mathematical concept has two properties: the first property is its capability of containing internal characteristics, inviting Bernstein's' (2000) hierarchical knowledge structure into elaboration. That is to say, one mathematical concept and its internal characteristics are hierarchically organised with the mathematical concept subsuming its internal characteristics. For example, based on the research data selected in this present study, "Pythagoras' Theorem" and "Introduction to Deductive Geometry" are hierarchically organised with "Introduction to Deductive Geometry" subsuming the internal characteristics of "Pythagoras' Theorem". The second property of one mathematical concept is its capability of connecting with its neighbouring mathematical concepts, inviting Bernstein's (2000) horizontal knowledge structure into elaboration. That is to say, one mathematical concept and its neighbouring mathematical concepts are horizontally organised where they are connected as incommensurable individuals. For example, the relationship between “Pythagoras’ Theorem” and “Quadrilateral” are neighbouring mathematical concepts. but incommensurable to each other. To be more specific, from a knowledge construction perspective, “Pythagoras’ Theorem” and “Quadrilateral” are distinctive on the one hand and share similar geometric and algebra properties on the one hand.

When recontextualisation was proposed (Bernstein, 1990), recontextualised relationships are identified between different linguistic phenomena, where the predominant semiotic resources in constructing discourses are solely linguistic resources (the use of verbal language). With the development of multi-modal and multi-semiotic studies, recontextualisation as a theoretical foundation underling the delocation and relocation of information between different discourses could be extended to include the recontextualisation between different modalities, while different modalities are constructed through different semiotic resources. The most typical incidents for the recontextualisation between different modalities will be bi-modal image-textual resources, where one modality is composed of linguistic resources and the other is composed of visual imagery. The underlying relationship between these two modalities will be recontextualisation within which the current streams underline the visual-verbal synergy, such as the work by Martinec and Salway (2005) and Royce (1998, 2002 & 2005). In their work, the visual-verbal relationship has been understood as visual resources elaborating, extending or enhancing the verbal resources, given the situation that visual resources are dependent on the verbal resources. In this study, we have to admit that what they have identified are types of recontextualisation within bi-modal texts, but not exhaustively so. The recontextualisation within bi-modal texts is far more complex. Drawing from the recontextualisation, different semiotic resources are co-dependent from an inter-textual perspective, but more importantly, each of the modalities has evolved its own way of meaning making. A synergistic perspective only considers how bi-modal text as the outcome, displayed the meaning as a combined effort by both modalities. A lot of hidden meaning has been dismissed when the only focus has been placed on the synergistic meaning between modalities. A recontextualisation perspective investigates multi-modal/multi-semiotic phenomena from two aspects. The first aspect is to see what type of features have undergone the process of recontextualisation, involving a dialogical perspective between different modalities. The second aspect is to investigate what types of features have remained and have not been recontextualised. This perspective is for the purposes of the underlying potential left unchanged.

Speaking of the relationship between knowledge storage and recontextualisation, Hasan and Butt (2006) reinterpret Bernstein's (2000) knowledge structure, arguing that forms of knowledge and forms of discourses are mutually informative of each other. This notion has been undertaken here in this study. The central argument of this study is to understand the mutually-informative relationship between knowledge structure and semiotic resources. As has been revealed in the analysis in Chapter Five and the findings in Section 6.1, knowledge structure could infer the lexical relationship such as taxonomic relationship, the activity sequences and implication sequences. Conversely, once the knowledge structure has not been explicitly offered by the discourse providers such as textbook writers, police makers and teachers, or has not been fully recognized by the students, the strategies developed in this study could be helpful in underlining the knowledge structure. These strategies are developed as linguistically empowered through deconstructing the semiotic resources within each pedagogic item. The relationship between different mathematical concepts could be reflected as the relationship between different lexical items. Each pedagogic item is an independent instance of realising the knowledge. This study has confirmed Hasan and Butt's (2006) argument through reconciling recontextualisation with reinstantiation. To be more specific, Bernstein's (1999, 2000) understanding of knowledge structure could be investigated from a linguistic approach. In this study, reinstantiation has been proposed as the innovative research model through which research data in the present study have been investigated.

Ideational commitment is selected as the baseline built upon which the similarities and differences between different instances of Pythagoras' Theorem are described. The ideational (re)instantiation is concerned with how the taxonomic relationship between different types of experiential features is constructed and how the logical relationship between these features is established. Drawing from Martin's (2007) statement, the central focus of ideational (re)instantiation is about knowledge relationship and is about how the commitments of knowledge relationship differ from one instance to another.

In sociological terms, knowledge is the baton being relayed and recontextualised. In social semiotic terms, the semiotic resources are undergoing reinstantiation with each instance of knowledge committing their own level of knowledge, demonstrating its own ways of making meaning. To some extent, this independence of each instance does not mean that different instances are isolated. There are still connections between different instances. Their connections have been argued primarily from an inter-textual perspective, addressing how they correspond with each other. In this study, the correspondence between different instances concerned in this study is visualized in the curriculum ecology outlined in Figure 3.5 and again in Figure 4.1.

More work is needed to understand how ideational meaning could be bridged with the knowledge structure. The approach taken in this study is a reconciled version of both recontextualisation and reinstantiation with support drawn from a multi-semiotic approach such as the work by O'Halloran (2005, 2007b).

Recontextualisation is an area worth more careful and theorized work. The current approach is essentially a social semiotic one where knowledge has been instantiated as different instances within the scope of a co-related system in the curriculum of mathematics, the curriculum ecology. Reinstantiation has been reconciled with recontextualisation.

In the field of education, the norms of recontextualisation are regulated by the social purposes of the designated discourse. For example, each of four types of pedagogic discourses, namely the syllabus (EDB, 1999), curriculum guideline (HKEAA, 2007), textbook (Wong and Wong, 2007) and examination paper

(HKEAA, 2012), have all been composed with their social purpose. This interpretation is argued from a broad perspective, acknowledging for each educational discourse in the curriculum ecology, that the norms for the semiotic construction of one particular discourse are essentially regulated by the social function co-related with it. In terms of different instances of the representation of Pythagoras' Theorem investigated in this study, the representation of this knowledge in each type of pedagogic discourse has been influenced by the context within which it was situated.

Based on the research data investigated in this study, four social purposes were revealed. First, the social purpose of the conceptual tables and diagrams in the syllabus (EDB, 1999) was to provide the knowledge structure between different mathematical concepts at the HKDSE level. Second, the social purpose of the instructional guideline in the curriculum guideline (HKEAA, 2007) was to enlighten teachers devising teaching strategies for future oral pedagogic discourse (such as classroom teaching). Third, the social purpose of the Theorem in the recommended textbook by Wong and Wong (2007) was to standardize the introduction of mathematical concepts, providing not only the internal characteristics within this mathematical concept but also the range of components to enhance the semiotic complex within mathematics. Fourth, the social purpose of the assessment tasks in the examination paper (HKEAA, 2012) was to assess the students' understanding of mathematical concepts with limited, vague and implicit ways of meaning making.

Each type of pedagogic discourse analysed in Chapter Five had its own way of making meaning; however, these ways of making meaning were also highly dependent on the contexts in which they were situated. The social functions of the syllabus, the curriculum guidelines, the textbooks and the examination papers allowed the recurrent patterns of meaning making that were suitable for each pedagogic discourse.

Extending the understanding from mathematics to other disciplines, this study could be treated as a stepping-stone for more projects of an interdisciplinary nature to be undertaken. These new emerging projects could be understood from the perspective of recontextualisation through which the knowledge and semiotic resources in one discipline are de-contextualised and recontextualised in other disciplines. These future research orientations are beyond the scope of this study, but the reconciliation of recontextualisation and reinstantiation explored and developed in this study will likely be helpful in understanding interdisciplinary linkage.

The curriculum ecology produced in this study is a model tentatively recommended to account for the discursive relationship between different pedagogic discourses within and between different disciplines in the education system. This model is mobilised with the help of recontextualisation. Ideally, this model could be of interest for education stakeholders generally. Therefore, this theorized model in designing the curriculum could be utilised for capturing the meaning-making process in different subjects. This model still needs more development based on the prototype provided here.

Many of the discussions in the analysis provided recontextualisation with an applicable linguistic theory therefore the discursive instances of knowledge representation could be investigated with evidence drawn from the linguistic approach. To reiterate here, language is the primary resource for making meaning, and knowledge is instantiated in language in the first place (Halliday & Matthiessen, 1999, p. 3). This understanding gives rise to the incorporation of ideational meaning as the primary parameter in investigating the various ways in which the same knowledge has been instantiated differently in a series of co-related mathematical discourses.

6.4 The Decontextualisation of Knowledge

After considering the norms of recontextualisation in Section 6.3, this section focuses on the decontextualisation of knowledge. To avoid the existing bias that once we know the language, we know exactly what is going on behind the language, the position held by this study is to treat knowledge behind the language as something not transparent to everyone (such as students and low level learners). What they could do is to deconstruct the concealed knowledge based on the semiotic resources available. Knowledge and its internal characteristics are hidden behind the semiotic resources we confront. Through a linguistic approach, the internal knowledge structure could be foregrounded. The linguistic approach we undertake here is a social semioticly informed one: concentrating on the Ideational metafunction in particular because as considered by Martin (2008), the content knowledge is by and large construed through ideational meaning. The specific framework within SFL that this study started from is a very specific point: the notion of (re)instantiation and its analytical moedel: ideational meaning commitment. This specific point underscores the differentiation in committing different experiential content (participant, process and circumstance) when the same knowledge corresponds with different instances alongside the system of curriculum. In this study, the system of curriculum is termed as curriculum ecology on which the relationship between different pedagogic discourses are treated as the logo-genetic recontextualisation with one pedagogic item is a recontextualised output of another. This process is metaphorically interpreted as a relay and our point of departure that initiates the relay is knowledge. Knowledge is the baton undergoing decontextualisation and recontextualisation. In the process, the associated pedagogic discourses are reformulated into different semiotic constructions. The way in which a pedagogic discourse appears in one way not the others has been intimately regulated by the type of discourse it its situated, representing the field of activity it stands for. To extend the field of activity that the specific pedagogic discourse bridges with, the types of tenor, the types of mode associated with this pedagogic discourse have all been prepared for a reproduction of the recurrent patterns that the certain pedagogic discourse instantiates.

6.5: Meaning Instantiation and Commitment

The present study selected instantiation as the parameter to explore, however, it could have shade light on the deeper understanding of stratification. Stratification focuses on the acceleration of meaning within a specific discourse, developing from the most internal structure of phonology and graphology, advancing to lexical-grammatical level and registerial level, and terminating at the generic level. Its theoretical foundation is to lay down an ontological development (vertical parameter) of a specific instance within a system. Instantiation, on the other hand, looks into the logogenic progression of meaning (horizontal parameter) by focusing on how different instances are differentiated with each other, are bridged with each other and are informed by each other where these instances are grouped under the same system of meaning. Each instance is a specification of the system, fostering its own way of meaning making. In this regard, the theoretical position held by instantiation could be subdivided into two steps: first, to understand how individual instance make meaning in their own domain and section, to understand how different individual instances are correlated with reference to their degrees of instantiation of the three metafunctional meanings, namely ideational meaning, interpersonal meaning and textual meaning. Following the systemic functional linguistic tradition, language and other semiotic resources are ways of making meaning where these three metafunctions could be proposed to underline the meaning making process from a trinocular perspective: how the field is construed, how the tenor is enacted and how the mode is constructed. Instantiation therefore, could be foregrounded with respects to the trinocular perspective.

This study takes the ideational meaning construction as the focus because the objective of this study is to underline the relationship between knowledge and representation. In terms of knowledge, issues such as technicality, lexical relationship, taxonomic relationship, generality and activity sequences etc., are the playground where knowledge structure intimately intersects with semiotic resources.

6.6 The accumulation nature of mathematics

Mathematics is the area where algebra laws, equations, mathematical theorems and the like at the lower level are treated as “prerequisite knowledge” at the higher level, suggesting constant accumulation of knowledge. The contrast between “lower level” and “higher level” could be projected as the two opposing poles on the continuum between primary grade one to senior secondary three in the school education system. Mathematical knowledge, as has been elaborated in the analysis, is instantiated in mathematical text. From the perspective of readers, one mathematical text could be subdivided into two parts: that which has already been learned and that which has not yet been learned. For example, the EDB has the institutional power to implement the mathematics curriculum and in sequencing the appearance of different mathematical concepts. The textbook producers follow the sequence determined by the EDB (1999) and HKEAA (2008) and produce their textbooks in accord with the guidelines. The same strategy has also been adopted by different schools whose sequential order in teaching the mathematical concepts are more or less the same, and in accord with the sequence underlined by the EDB (1999) and the textbooks (e.g. Wong & Wong, 2009). That parallel relationship in advancing from "learned" content to "unlearned" content is reflective of the education system from a broad perspective. With reference to individual students, the balance has been individualised as the dialogue between "known" and "unknown", which addresses the highly personalized learning experience at the students' conceptual level. The pace of movement from the "unknown" to the "known" is highly dependent on the students' own understanding and efforts. That is why autonomous learners are always more advanced than are non-autonomous students. They deliberately speed up the transaction from "unlearned" to "learned", prior to the actual learning experience, which their peers are yet to experience.

How could we guarantee a student will "know everything that they have learned?" is a huge question that is beyond the scope of this study. However, what this study could do is to ensure that what the students have to learn is explicit so that they could bridge what is being required with what they have already learned. This effort, as the central focus of this study is to understand the knowledge structures in mathematics from a linguistic perspective. We, as education linguists could guide our students as well as other education stakeholders to comprehend fully the mathematical discourse by deconstructing it into manageable pieces, which have the potential to be recontextualised. Recontextualisation argued here in this study is a bi-directional process. By saying bi-directional process, one mathematical discourse could be recontextualised into others. It could also be recontextualised from others. In mathematics, the pieces of information that could be both recontextualised from other discourses and/or recontextualised into other discourses is what I will term, mathematical cues. Mathematical cues will be discussed in the following section.

6.7 The nature of cues in understanding mathematical discourse

In the process of understanding how each independent component commits its own ideational meaning, mathematical discourses undergo deconstruction through which the mathematical discourses could be broken into different pieces of information while each component commits a certain degree of ideational meaning. Speaking from a meaning construction perspective, these pieces of information are textually organised resources appearing as mathematical verbal language, mathematical symbolism or visual images or as a semiotic complex of their intersection. Each piece of information in its own right has the potential to be textually co-related with the ones in other mathematical discourse, for example, through linguistic cohesion such as repetition.

From a broad perspective, instances analysed in the Syllabus (1999), curriculum guidelines (HKEAA, 2007), textbooks (e.g. Wong & Wong, 2009) and the examination paper (HKEAA, 2007), have one thing in common: the knowledge of Pythagoras' Theorem. They are cohesive because of the repetition of the technical term "Pythagoras' Theorem" between Syllabus (1999), curriculum guidelines (HKEAA, 2007) and textbooks (e.g. Wong & Wong, 2009), and the repetition of the same geometrical image between textbooks (e.g. Wong & Wong, 2009) and examination papers (HKEAA, 2007).

From a narrow perspective, based on Figure 5.38, the repetition of the letters "a", "b" and "c" in the verbal description, the symbolic equation and the visual imagery enables these three components to be linguistically connected with each other.

Both from a broad perspective of understanding the curriculum structure and from a narrow perspective of understanding the type of instance, repetition of linguistic, symbolic and/or visual resources is crucial in bringing different pedagogic discourses together and in bringing together different components within an instance, enabling the recontextualisation of mathematical laws, theorems, or algorithm from one discourse to another.

It must be noted here that the devices either from a broad perspective or from a narrowed perspective are termed “cues”. In mathematics, I term them “mathematical cues”.

Mathematical cues are facilitated by lexical cohesion. This facilitation of the lexical cohesion will lead to an inclusion of the conceptually required knowledge into the solution of the issues at hand by incorporating “the whole bodies of knowledge” (Linell, 1998, p. 156) into the newly encountered mathematical discourse. For example, in order to solve the examination question in Figure 5.29, the available semiotic resources utilized in Figure 5.38 to elaborate the knowledge of Pythagoras’ Theorem in the textbook will be the cues functioning to help solve the question. In particular, visual imagery in Figure 5.38 is repeated in the assessment task in Figure 5.29. These two instances, one in the textbook and the other in the examination paper, are bridged due to their common visual image.

Speaking from a sociological perspective, cues are the catalyst in the process of recontextualisation within which the knowledge is included, delocated and relocated from one discourse to another. Speaking from a linguistic perspective, cues are the linguistically repeated parts shared by different discourses. This notion of linguistically repetition has been extended to include all ranges of mathematical semiotic resources in this study. That is to say, the repetition of verbal resources, mathematical symbolism and visual imagery could all be the potential resources in constructing the cues for the purposes of recontextualisation of knowledge between different texts.

Cues therefore are crucial and the understanding and elaboration of the cues helps to more clearly understand the knowledge representation when our major focus on this issue is linguistically oriented.

6.8 Intersemiotic nature of mathematics

O'Halloran's work (2007b) discusses the complexity of the multi-semiotic interplay identified in mathematical discourse. The combination of different semiotic resources proposes "a major challenge for multimodal discourse analysis" (O'Halloran, 2007, p. 89). This challenge is to understand how different semiotic resources interact and coordinate with others. In mathematics, regarding the meaning-making process, the central issue is to understand the interplay between language, mathematical symbolism and visual imagery or in other words, to understand the "intersemiotic nature of mathematics" (O'Halloran, 2005, p. 16). Building upon the social semiotic theory by Halliday (1978), the inter-semiosis proposed by O'Halloran (2005, 2007a) is a research framework designed to investigate the inter-semiotic nature of mathematics "where meaning is the product of linguistic, visual and symbolic choices" (O'Halloran, 2007a, p. 79).

In mathematics, the situation where different semiotic resources "function intrasemioticly as closed systems" (O'Halloran, 2005, p. 165), exists only in theory. In practice, "semiotic resources have evolved to be used in conjunction with other semiotic resources" (O'Halloran, 2005, p. 165). Therefore, to see mathematical discourse from the inter-semiotic perspective will be holistic to the understanding of the system networks applied.

According to O'Halloran (2007), inter-semiotic mechanism accounts for the understanding of how different metafunctional meanings "played out across choices from the semiotic resources across the different levels" (p. 89). Inter-semiosis gives rises to the recontextualization of metafunctional meanings through the semiotic choices. Re-contextualization corresponds with Bernstein (1990) who explains that semantic shifts occur according to contextualizing principles, which "relocate, refocus and relate to other discourses to constitute its own order and orderings" (Bernstein, 1990, p. 184). Here in mathematics, the recontextualization of meanings indicates the divergence in the semiotic choices where different semiotic resources and different combinations of semiotic resources could be relocated, refocused and related in constituting different representations of mathematical knowledge. This divergence in the semiotic choices involves the process of "meaning compression" (Baldry & Thibault, 2009, p. 19) where information in one cluster is recontextualized in other clusters. This approach echoes the research paradigm of social constructionism of knowledge where knowledge and its representation have been separated. Yet, it also suggests a direction of analysis where knowledge as the information encapsulated in a certain form of discourse, has the potential to be recontextualized into different discourses represented in different semiotic resources. This meaning compression process could be applied to mathematical discourse (O'Halloran, 2007) while the inter-semiotic mechanism facilitates the understanding of how meaning is made in each representation of mathematical knowledge from a multi-modal and multi-semiotic perspective.

6.9 The contribution that this study could present to the education stakeholders

Throughout the study, a central issue repeatedly brought up is to address the relationship between different mathematical concepts at a conceptual level. The proposition suggested by this study is a dialogue between Bernstein's (1999 and 2000) sociological approach of knowledge relationship and Halliday and his colleagues' work on language as the meaning-making resources (e.g. Halliday, 1973, 1978, 1994; Halliday & Matthiessen, 1999, 2004; Halliday & Martin, 1993). In this study, language and other semiotic resources are the tools underlining how knowledge structures are established through foregrounding the lexical relationship between different mathematical concepts.

The most straightforward way of underlying the knowledge structures is the use of a textual layout. For example, the tabular taxonomy and flowchart in EDB (1999) directly translate the conceptual representations between different mathematical concepts into the knowledge structures between them as shown in Section 5.2.

As for the curriculum guideline, knowledge structures between different mathematical concepts are embedded, not in their lexical relationships, but in the activity sequences and nuclear relations as shown in Section 5.3.

The investigation of Pythagoras' Theorem in this study reveals the complexity of how knowledge could be represented in a series of co-related pedagogic documents. In this study, Pythagoras' Theorem is the focus where each instance of knowledge representation arises from this mathematical concept. Of course, we can investigate other mathematical concepts. However, focusing on Pythagoras' Theorem has already justified the research objective of this study. The objective of this study is concerned with how a mathematical concept has been represented in a series of co-related mathematical documents and how these different representations are co-related and differentiated from each other. The research findings drawn from the analysis of Pythagoras' Theorem could be replicated in the investigation of other mathematical concepts.

To extend the output generated from this research and to inform future research to be undertaken in a wider scope of all mathematical concepts, the ways through which this study has been conducted could be replicated to conduct similar research in other disciplines, addressing the significance of incorporating the dialogue between social semiotics and sociology into the educational related projects where the primary focus is to treat language (and also other semiotic resources) as the primary resources in making meaning. Knowledge together with the internal and external knowledge structures could be inferred based on the semiotic resources at hand. The work that this study has investigated is determined to transcend the invisible conceptual knowledge that is central in the field of education into salient linguistic (and of course semiotic) resources. Based on these resources, knowledge structure could appear to be less vague and abstract. For example, the representation of the knowledge of "cloning" in biology textbook is achieved through logical-sequential relationship between different activities (Xia, 2015). Below is the text taken from the biology textbook (Ho, 2012, p. 48).

> In the cloning of a sheep, British scientist Ian Wilmut (1944-) and his team collected epithelial cells from the mammary gland of an adult white-faced donor sheep. The cells were allowed to fuse with enucleated eggs (eggs with nuclei removed) from an adult black-faced recipient sheep. The fused cells were then grown in culture for six days so that embryos could develop. The

> embryos were then implanted into the uterus of an adult black-faced sheep (the surrogate mother). The sheep became pregnant and the fetus developed.
>
> (Ho, 2012, p. 48)

Activities identified in the text such as: the collection of epithelial cell, the fusion with enucleated eggs, the growth in culture, and the implantation into the uterus are the prerequisite steps before "cloning" could succeed. This relationship at linguistic level could be viewed from a knowledge construction perspective, treating these prerequisite steps as the prerequisite knowledge hierarchically organised with "cloning".

This study provides the education stakeholders with recontextualisation principles that could help them focus more acutely on how language has performed as a crucial meaning-making resource, and how this resource could be used to reveal the concealed knowledge structure in mathematics. Building up to the curriculum ecology sketched in this present study, similar logo-genetic research in the field of mathematics and in other disciplines could follow the analytical blueprints and the research frameworks outlined in this study.

6.10 Pythagoras' Theorem or "Gou Gu Theorem": a linguistic exploration

The significance of selecting Pythagoras' Theorem as the focus of this study is presented in Section 4.5. One of the significance is to provide an exploratory answer to the Grand Needham Question based on a linguistic exploration of Pythagoras' Theorem.

Pythagoras' Theorem which is named after the great Ancient Greek philosopher: Pythagoras is regarded as the "most famous statement in all of mathematics" (Maor, 2007, p.preface). At around 530 BC, Pythagoras was the first one in western history that "came up with the idea establishing this geometric relationship" (Posamentier, 2010, p.24) between hypotenuse and other two sides in a right-angle. At around 300 BC, Euclid documented the findings of Pythagoras in his masterpiece of mathematics: Elements and named this theorem as "Proposition 47 in Book I of Elements" (Maor, 2009, p.xi). At around 450 AD, Proclus, the Greek philosopher, offered a comprehensive commentary version on Euclid's Elements and renamed Proposition 47 as Pythagoras' Theorem. Since then, Pythagoras' Theorem as a technical name was widely used in mathematics and other disciplines.

However, whether this theorem should be named after Pythagoras was questioned some Chinese scholars (cf: Cheng, 1951; Zhang, 1951) since they believe that this theorem first appeared in Ancient China at around 1120 BC, nearly six hundred years earlier than Pythagoras' finding. The most classic mathematics book: Zhou Bi Suan Jing (*The Mathematics Book of Zhou Dynasty*) documented the instance of this theorem. Needham (2005) translated the Chinese version as "Thus cut a rectangle (diagonally), and make the width (kou) 3 (units) wide, and the length (ku) 4 (units) long. The diagonal (ching) between the (two) corners will then be 5 (units) long" (2005, p. 22).

Needham (2005) further suggested that "the Chinese proof of the Pythagoras theorem was indeed a proof" (p. 103), a proof of Pythagoras' Theorem applying to the specific instance of the "3-4-5 sided triangle" (Maor, 2007, p.63).

Based on this specific 3-4-5 instance, some Chinese scholars (c.f. Cheng, 1951; Zhang, 1951) insisted that "Gou-Gu Theorem" should be the right name rather than Pythagoras' Theorem to represent the relationship between different sides in a right-angled triangle.

Drawn on the analysis in Chapter Five, the 3-4-5 sided triangle provided in Zhou Bi Suan Jing is highly contextually dependent, focusing on one right-angled triangle only. The western version which is the one taken from the textbook, on the other hand, could be expanded to account for a wider range of application with the help of irrational numbers: a, b and c. Therefore, the Chinese version is not a theorized one and lacks exploratory power relying on which more similar data instances could be accounted for. In this respect, I think it is correct to name this theorem as Pythagoras' Theorem.

The comparison between different versions of the same theorem opens up an exploration of the "Needham Grand Question": why had modern science not developed in Chinese civilisation? (Needham, 2005). Based on the comparison, my answer to this question is a linguistic one: it is because that as the foundation for technological development, manuscripts written in Chinese did not favour a writing style where knowledge has been theorized as what Pythagoras did but implicated the knowledge as concrete examples.

Chapter Seven – Conclusion

7.1 Introduction

The purpose of this study has been to understand how knowledge is represented differently in a series of co-related pedagogic discourses. The scope of this study was specified within the domain of mathematics because, as mentioned at the outset in this thesis, "Mathematics is the queen and servant of science". The better understanding of the meaning-making process in mathematics will enrich the understanding of other disciplines. Due to the limitations of time and space, this study selectively analysed one key mathematical concept at the HKDSE level. This mathematical concept is Pythagoras' Theorem. Drawing from the curriculum ecology outlined in Chapter Three, pedagogic discourses viz. the syllabus (EDB, 1999), the curriculum guideline (HKEAA, 2007), mathematical textbooks (e.g. Wong & Wong, 2009) and the examination paper (HKEAA, 2012) have been selected as the sources of data from which instances of the representation of Pythagoras' Theorem have been examined. It must be noted that the selected pedagogic discourses could be categorized as non-classroom pedagogic discourses because, compared with classroom pedagogic discourses, the pedagogic discourses examined in this study are not produced in classroom settings. They are composed by either an authority stream (such as the Education Bureau of Hong Kong, the Hong Kong Examination Assessment Authority) or a commodity stream in the form of written texts (such as from the shortlisted commercial publishers discussed in Table 4.2). Bernstein (1990) confirms the prominence of pedagogic discourse outside the classroom settings because these pedagogic discourses are a major impetus in achieving pedagogic purposes. Bernstein's (1990) confirmation gives rise to the practical consideration of this study; that is, the analysis of pedagogic discourses outside the classroom settings is also valuable in understanding the discursive meaning-making processes in the school system. As for the theoretical consideration of this study, recontextualisation has been proposed as the driving force mobilizing the curriculum ecology. Based on the analysis in Chapter Five, recontextualisation, which is a sociological consideration of the change between different pedagogic discourses, has been merged with reinstantiation, a linguistic model for modelling the relationship between system and instance. This merge, as has been revealed in the analysis, has provided the discussion of the relationship between knowledge and representation with a workable understanding. That is to say, each representation of Pythagoras' Theorem has been understood as an independent instance of the representation of the abstract knowledge of Pythagoras' Theorem in the real world. The differences of the representations are controlled by the contexts in which they are situated.

This chapter concludes this study through summarizing the major content rendered by each chapter. The limitations of this study, together with possible future research areas, are provided as well.

7.2 Summary of this study

Chapter One provides the background to this study. The significant role of mathematics in both the school systems in Hong Kong and the development of contemporary science have been provided. Chapter One also outlines the structure of this thesis, providing the overview of each chapter.

Chapter Two discusses key studies in understanding mathematical knowledge, knowledge structure and the nature of mathematical discourse. In this present study, mathematical knowledge is interpreted as abstract and invisible mathematical concepts. The relationship between different mathematical concepts could be understood as a connected network structure whose underlying principle of organisation is Bernstein's (2000) horizontal knowledge structure and hierarchical knowledge structure. The representation of mathematical knowledge requires mathematical semiotic resources. According to O'Halloran (2000), three key semiotic resources involved in realising mathematical knowledge are verbal language, mathematical symbolism and visual imagery.

Chapter Three suggests a social constructivism perspective in understanding knowledge and its representation, empowering the synthesis rendered in Chapter Two. Recontextualisation and re-instantiation are proposed as the two key theoretical considerations of mathematical discourses, compared and merged to suggest how the meaning-making process in mathematical discourses could be achieved. The nature of curriculum ecology is offered in order to justify that this model could be understood as the default mechanism on which the school system is mobilised.

Chapter Four covers the research methodology in which the research questions, the research data and the justification for the research data is offered. Part of the curriculum ecology is under examination. This part is the non-classroom data that also plays a crucial role in the education system and typically receives little attention. Pythagoras' Theorem is selected as the focus of analysis with instances related to Pythagoras' Theorem in the contexts of the Syllabus (EDB, 1999), the curriculum guideline (HKEAA, 2007), the textbook (e.g. Wong & Wong, 2009) and the examination paper (HKEAA, 2012). A qualitative analytical approach is designed with recontextualisation being merged with reinstantiation. Ideational commitment is the key research model in understanding how Pythagoras' Theorem has been variously instantiated in different instances of representation. A blueprint for analysis is provided.

Chapter Five comprises the data analysis and discussion. Each instance of Pythagoras' Theorem selected in this study was examined following the blueprint outlined in Chapter Four. The relationship between knowledge structure and semiotic construction was argued logo-genetically, following a continuum consisting of the Syllabus (EDB, 1999), the curriculum guideline (HKEAA, 2007), the recommended mathematical textbook (Wong & Wong, 2009) and the examination paper (HKEAA, 2012). A comparison of instances taken from these pedagogic discourses has also been provided, confirming that the semiotic construction for each instance of Pythagoras' Theorem is highly dependent on the context within which it is situated.

Chapter Six consists of the findings and discussions of aspects of this study. The relationship between knowledge structure and semiotic construction, the nature of mathematical discourses, the nature of recontextualisation and the implications that this study may have for education stakeholders have been provided.

7.3 More work required to understand recontextualisation

Recontextualisation is one of the key theoretical considerations in this study. The scope of recontextualisation is broad. It encompasses nearly every instance where there is a change from one context to another. Recontextualisations is a term that has been used broadly but lacking theoretical underpinning. In this study, the theorization of recontextualisation will empower our treatments towards discourse analysis within an understanding of the default characteristics in the process of recontextualisation with an involvement of more salient linguistic resources. Several existing theoretical considerations bear the underlying characteristics of recontextualisation. For example, translation is a form of recontextualisation where changes take place between different languages. Visualisation is a form of recontextualisation where change occurs in the transformation from non-visual resources to visual resources. Digitalisation is a form of recontextualisation. It is an updated version of visualization where changes take place at a more technical level. Verbalisation is also a form of recontextualisation where changes take place between the transformation from the non-verbal resources to the verbal resources. In the study of the film industry, salutation, which is a type of filmic strategy, is also a form of recontextualisation where the plot, characters, lines, or scenes from a classic film are reinstantiated into the current production. Still within the film industry, the anatomy of salutation will be plagiarism. It differs from salutation in terms of the subjective malicious intent by the producer and/or the director. The debate between salutation and plagiarism in the film industry is controversial in terms of the legal issues involved; however, in terms of linguistic arguments, these two types of filmic strategies are types of recontextualisation.

Plagiarism also prevails in academia, differentiating itself from "citation". The potential for plagiarism is overcome by the use of correct citation and referencing. To keep this in mind, a typical citation uses quotation marks to indicate the quoted content. A typical citation is a hypotactic projecting clause within the cited part being included. The original author(s) functions like the speakers as in verbal clauses or as thinkers as in mental discourse. The omission of speakers or thinkers marks the plagiarism. Conceptualization is also a form of recontextualisation through which all the resources such as the linguistic description have been conceptualized, appearing as a concept. Nominalization is also a form of recontextualisation through which the process has been encoded as a noun.

Some pioneering work bears features of recontextualisation. For example, resemiotization, proposed by Iedema (2003), is informed by recontextualisation through outlining the reformation relationship between verbal language and visual images. According to van Leeuwen (2013), re-semiotization which is textually oriented, focuses mostly on how textual features are reformed between different modes. Informed by recontextualisation and reinstantiation, Tang (2013) describes how different visual images reshape each other, highlighting the textual perspective of different modes in science education. Inter-textuality, proposed by Bhatia (2008) operates at a text level, through arguing the textual relationship within one single document. The notion of Inter-discursivity (Bhatia, 2010; Lam, 2013) extends inter-textuality from a single text to different texts by arguing how dialogical relationships are established between different texts at the textual domain.

Re-semiotization (Iedema, 2003), re-representation (Tang, 2013), inter-textuality (Bhatia, 2008) and inter-discursivity (Bhatia, 2010; Lam, 2013) are technical terms dealing with the meaning making process from a textual perspective. Compared with these theoretical considerations, recontextualisation and reinstantiation also work at the ideational perspective and interpersonal perspective. Examples are those such as the work by Rose and Martin (in press) for the understanding of the instantiation of ideational meaning in school contexts and Kong's (2008) work for incorporating recontextualisation to underline the commitment of different interpersonal meanings in different news reports regarding the real estate advertisements.

Regarding the scope of recontextualisation and reinstantiation, both of them could incorporate the dialogical relationship within one single text and between different texts (Linell, 1998) and account for the semiotic resources of language and other semiotic resources (Painter et al., 2013).

The phenomenon that recontextualisation accounts for the transition between different education practices is a sociologically-oriented approach proposed by Bernstein (1999). From a knowledge construction perspective, the point of departure includes knowledge in the relay between one education practice and another. In the field of education, recontextualisation has been reconciled with reinstantiation by SFL scholars (Rose & Martin, in press) to provide recontextualisation with linguistic evidence. The relationship between system and instances in the framework of reinstantiation could be reconciled with the relationship between the knowledge system and the different representation of knowledge in the framework of recontextualisation. Within these two frameworks, the smallest particle is the invisible knowledge. Language and other semiotic resources are applied to visualise the invisible knowledge. The reconciliation of recontextualisation and reinstantiation offers the opportunity to investigate the discursive ways of knowledge construction and representation available in education fields.

7.4 Recontextualisation and inter-disciplinary approach

The quote "Mathematics is the queen and servant of science" has foreseen a recontextualisation relationship between mathematics and other disciplines. That is to say, mathematical knowledge together with mathematical theorems and algorithms could be recontextualised into other scientific disciplines. For example, the classic Newtonian laws of motion (i.e. $f = ma$) is composed of both physical knowledge of motion, mathematical symbolisms (f stands for force, m stands for mass, a stands for time of acceleration) and a mathematical algorithm. Theorems could be identified in other scientific disciplines as well. More work could be dedicated to explore how the establishment of scientific disciplines has been influenced by mathematics from a sociological and linguistic approach in future.

7.5 Limitations of this study

As has been noted in Section 7.4, an inter-disciplinary approach in understanding the relationship between mathematics and other disciplines will enrich our understanding of the internal relationship within the education system at the HKDSE level. With respect to mathematics, which is the area under examination in this study, research data is selected relating to pedagogic discourses outside the classroom. In order to comprehend the complete meaning-making process in mathematics, classroom data such as teachers' talk, teacher-student interactions, lecture notes and other pedagogic discourse applied in classrooms will help to fill this gap. The inter-disciplinary approach and the inclusion of classroom data will be two research angles worthy exploring in future.

From a theoretical perspective, this study explores the representation of Pythagoras' Theorem with the help of ideational commitment and interrelates different levels of ideational commitment with the knowledge structure. More work could be conducted to understand what roles interpersonal meaning and textual meaning play in understanding the relationship between knowledge and representation. For example, how the knowledge is exchanged between teacher and student could be investigated from an inter-personal perspective, highlighting dynamic information exchange. As for textual meaning, future research could be conducted to understand what information has been thematised, placed at the theme position, and how this thematisation of information facilitates knowledge construction.

7.6 Concluding remarks

This study derives from the personal interest of the author in understanding how different texts in one curriculum scaffold the representation and progression of knowledge across one curriculum. A qualitative analytical approach is conducted in this study for the purposes of understanding how the same mathematical knowledge has been represented differently in a series of co-related pedagogic discourses.

To look at mathematical discourses from the perspective of instantiation "will not change the principles of the theory construction" (Halliday, 2004, p. 220). As for the present study, the underlying principles of Pythagoras' Theorem and other mathematical concepts have been in place for a long time. However, to look at different instances of the same mathematical concept will add new types of instantiation and hence “will broaden our conception of possible kinds of reality” (Halliday, 2004, p. 220). As for the present study, our conception of how Pythagoras’ Theorem has been represented differently between a series of co-related pedagogic discourses is articulated. There may be more types of instantiation with the involvement of more pedagogic discourses that have not been investigated in the present study (such as classroom discourses); however, the logo-genetic approach taken in the present study could mark a stepping stone on the basis of which new and similar studies could follow the path established in this study.

Focusing on mathematics does not mean the findings informed by this research could only be constrained to the field of mathematics. Rather, what this research is intended to argue is from the very rudimentary dichotomy between knowledge and representation. The findings in the mathematics could be extended to cover other curriculums. For example, each curriculum could cultivate its own curriculum ecology. Mathematics is the basis and foundation of all science subjects. Its basic principles, laws and theorems have already been constantly recontextualised into other subjects, advocating the long-standing interests in inter-disciplinary studies.

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Appendices

Appendix One: Three instances of Pythagoras' Theorem: p. 13, p. 23 and p. ANNEXX III in Syllabus (EDB, 1999)

Table A1.1: Teaching of Measures, Shape and Space Dimension

Key Stage 3 (S1-S3)	Key Stage 4 (S4-S5)
Measures in 2-Dimensional (2D) and 3-Dimensional (3D) figure	
• Estimation in Measurement (6) • Simple Idea of Areas and Volumes (15) • More about Areas and Volumes (18)	
Learning Geometry through an Intuitive Approach	
• Introduction to Geometry (10) • Transformation and Symmetry (6) • Congruence and Similarity (14) • Angles Related with Lines and Rectilinear Figures (18) • More about 3-D Figures (6)	• Qualitative Treatment of Locus (6)
Learning Geometry through a Deductive Approach	
• Simple Introduction to Deductive Geometry (27) • Pythagoras' Theorem (8) • Quadrilaterals (15)	• Basic Properties of Circles (39)
Learning Geometry through an Analytic Approach	
• Introduction to Coordinates (9) • Coordinates Geometry of Straight Lines (12)	• Coordinate Treatment of Simple Locus Problems (14)

Trigonometry	
• Trigonometric Ratios and Using Trigonometry (26)	• More about Trigonometry (29)

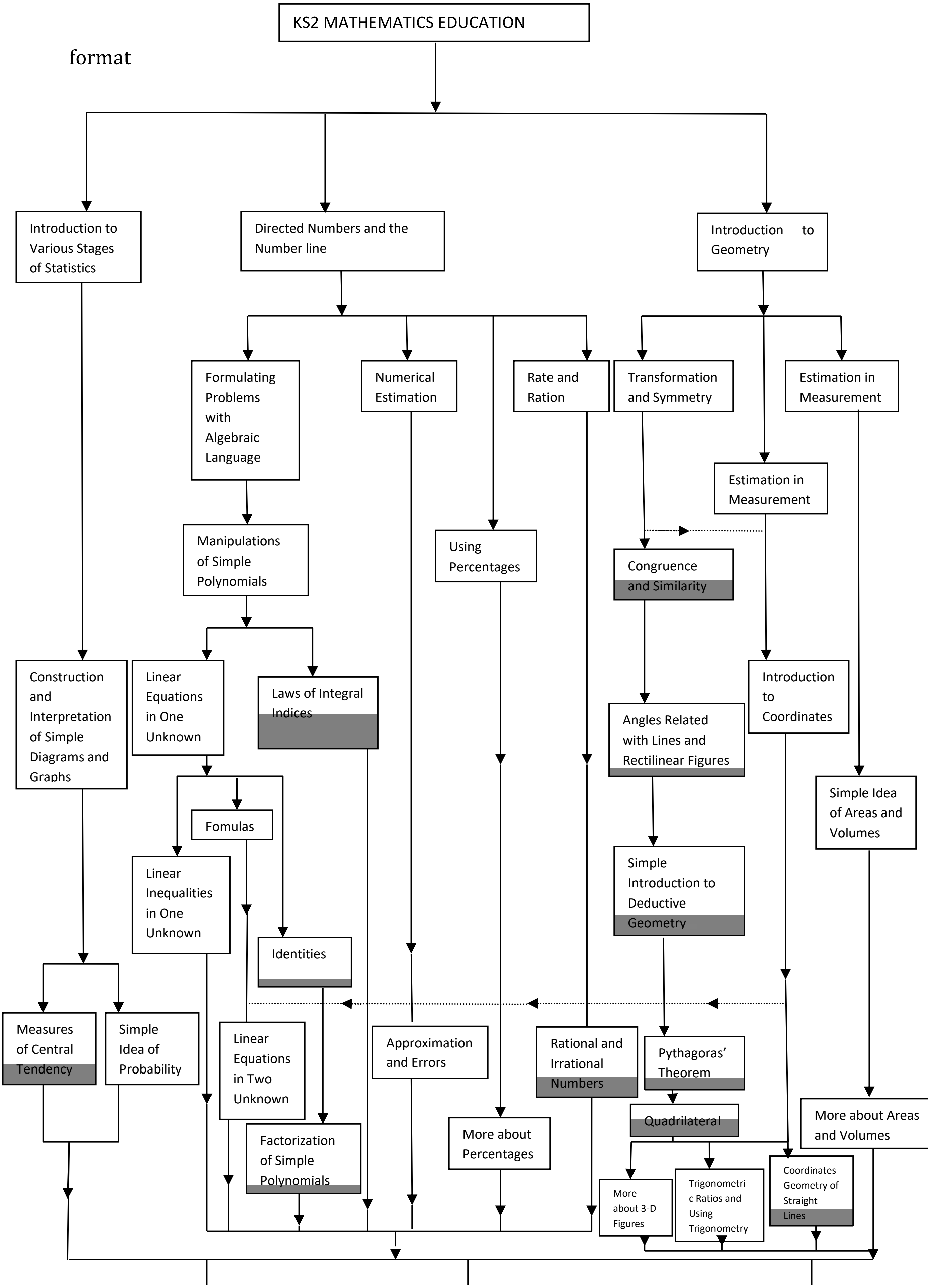
Table A1.2: Learning Geometry through a Deductive Approach

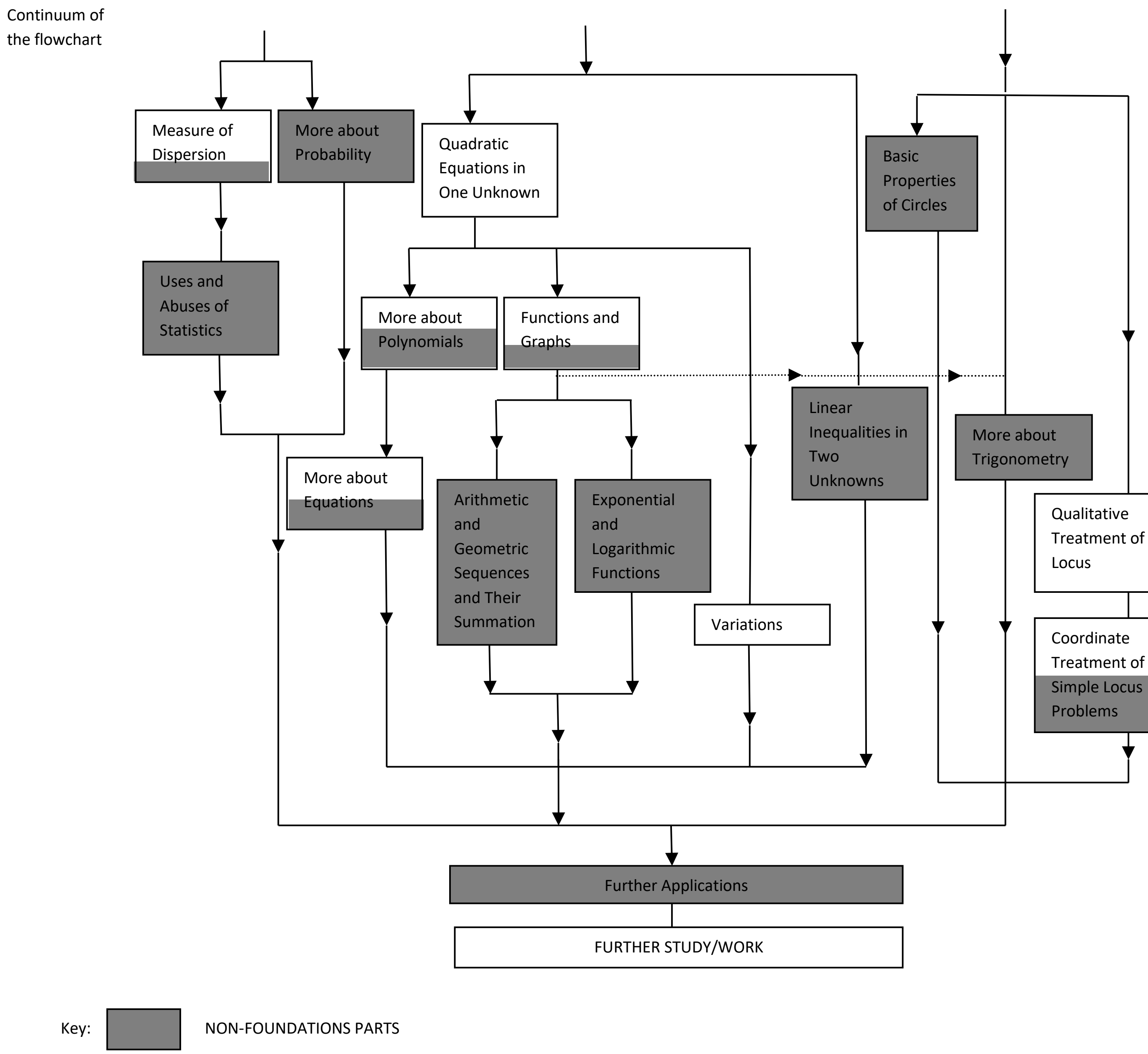
Unit	Learning Objectives	Suggested time ratio
Learning Geometry through a Deductive Approach		
Simple Introduction to Deductive Geometry	• develop a deductive approach to study geometric properties through studying the story of Euclid and his book- *Elements* • develop an intuitive idea of deductive reasoning by presenting proofs of geometric problems relating with angles and lines • understand and use the conditions for congruent and similar triangles to perform simple proofs • identify lines in a triangle such as medians, perpendicular bisectors etc. • explore and recognize the relations between the lines of triangles such as the triangle inequality, concurrence of intersecting points of medians etc. • explore and justify the methods of constructing centres of a triangle such as in-centre, circumcentre, orthocentre, centroids etc. • ** prove some properties of the centres of the triangle	27
Pythagoras' Theorem	• recognize and appreciate different proofs of Pythagoras' Theorem including those in Ancient China • recognize the existence of irrational numbers and surds • use Pythagoras' Theorem and its converse to solve problems • appreciate the dynamic element of mathematics knowledge through studying the story of the first crisis of mathematics • **investigate and compare the approaches behind in proving Pythagoras' Theorem in different cultures • **explore various methods in finding square root	8

Quadrilaterals	• extend the idea of deductive reasoning in handling geometric problems involving quadrilaterals • deduce the properties of various types of quadrilaterals but with focus on parallelograms and special quadrilaterals • perform simple proofs related with parallelograms • understand and use the mid-point and intercept theorems to find unknowns	15

Note: (i) The objectives with asterisk (**) are considered as exemplars of **enrichment topics.** The objectives <u>underlined</u> are considered as a **non-foundation** part of the syllabus. (ii) say something about Table A1.1 differing from Table xxx (find the table number in the body).

format


KS2 MATHEMATICS EDUCATION
Introduction to Various Stages of Statistics
Directed Numbers and the Number line
Introduction to Geometry
Formulating Problems with Algebraic Language
Numerical Estimation
Rate and Ration
Transformation and Symmetry
Estimation in Measurement
Estimation in Measurement
Manipulations of Simple Polynomials
Using Percentages
Congruence and Similarity
Construction and Interpretation of Simple Diagrams and Graphs
Linear Equations in One Unknown
Laws of Integral Indices
Angles Related with Lines and Rectilinear Figures
Introduction to Coordinates
Simple Idea of Areas and Volumes
Fomulas
Linear Inequalities in One Unknown
Simple Introduction to Deductive Geometry
Identities
Measures of Central Tendency
Simple Idea of Probability
Linear Equations in Two Unknown
Approximation and Errors
Rational and Irrational Numbers
Pythagoras' Theorem
More about Areas and Volumes
Factorization of Simple Polynomials
More about Percentages
Quadrilateral
More about 3-D Figures
Trigonometric Ratios and Using Trigonometry
Coordinates Geometry of Straight Lines



Note: Mathematical knowledge is inter-related both within and across dimensions. It is important to illustrate all links in a flowchart. Strong links between learning units are shown in dotted lines. These lines are just for illustrations and are not meant to be exhaustive. Teachers should exercise their judgment in arranging the sequence of learning units with special attention to the prerequisite knowledge required. For example, students are required to have the pre-requisite knowledge in “Introduction to Coordinates” to solve “Linear Equations in Two Unknowns” by graphical methods.

Figure A1.1: Flowchart of Learning Units for Secondary School Mathematics Curriculum (adapted from EDB, 1999, p. ANNEX III)

Note: Mathematical knowledge is inter-related both within and across dimensions. It is important to illustrate all links in a flowchart. Strong links between learning units are shown in dotted lines. These lines are just for illustrations and are not meant to be exhaustive. Teachers should exercise their judgment in arranging the sequence of learning units with special attention to the pre-requisite knowledge required. For example, students are required to have the pre-requisite knowledge in “Introduction to Coordinates” to solve “Linear Equations in Two Unknowns” by graphical methods.

Appendix Two: One Instance of Pythagoras' Theorem in Curriculum Guidelines (HKEAA, 2007, p. 26)

This example illustrates some of the approaches and strategies that can be used in the Mathematics classroom.

Teaching one of the properties of the scalar product of vectors using the direct instruction, the inquiry and the co-construction approaches

Teachers may integrate various teaching approaches and classroom practices to introduce the properties of the scalar product of vectors so that the lessons can be more vivid and pleasurable. In this example, teaching one of the properties of the scalar product of vectors, $\mathbf{|a-b|^2 = |a|^2 + |b|^2 - 2\,(a \bullet b)}$, is used as an illustration.

In previous lessons, the teacher has taught the concepts of magnitudes of vectors and the scalar product of vectors using **direct instruction.** In this lesson, the students are divided into small groups to promote discussion, and the groups are asked to **explore** the geometrical meaning of the property. Here, the **inquiry approach** is adopted, with students having to carry out **investigations** with the newly acquired knowledge related to vectors. During the exploration, the groups may interpret the geometrical meaning differently. Some may consider one of the vectors to be a zero vector and get the above property; but others may relate it to the Pythagoras' Theorem by constructing two perpendicular vectors a and b with the same initial point. Hence, the hypotenuse is $|a-b|$ and $a \bullet b = 0$ and the result is then immediate. If some groups arrive at this conclusion, the teacher should guide them to discover that their interpretation is only valid for special cases. However, the geometrical meaning of this property is related to the cosine formula learned in the Compulsory Part. If some groups can find that the property is the vector version of the cosine formula, they can be invited to explain how they arrived at this geometrical meaning. If none of the groups can arrive at the actual meaning, the teacher may guide them to find it out by giving prompts. Some well-constructed prompts (or scaffolds), such as asking them to draw various types of triangles and find clues to connect |a-b|, a•b, |a| and |b| with triangles drawn, may be provided. The co-construction approach is adopted here.

After understanding the geometrical meaning, the result can be derived by applying the cosine formula learned in the Compulsory Part. The groups are further asked to explore alternative proofs. Here, the inquiry approach is employed. The groups may not think of proving this property with $|x|^2 = x \bullet x$ directly. The teacher may give some hints to guide them. In this case, the teacher and the students are co-constructing knowledge. If the students still cannot prove this property, the teacher can demonstrate the proof on the board using the direct instruction approach. Whatever methods the students use, they are invited to explain their proofs to the class. During the explanation, the teacher and student may raise questions and query the reasoning.

Appendix Three: Pythagoras' Theorem in Examination Paper (HKEAA, 2012, p. 2)

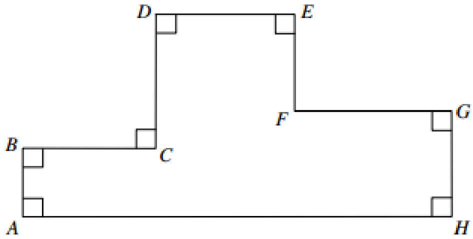
18. In the figure, $AB = 4$ cm, $BC = CD = DE = 8$ cm and $FG = 9$ cm. Find the perimeter of ΔAEH.

A. 60 cm

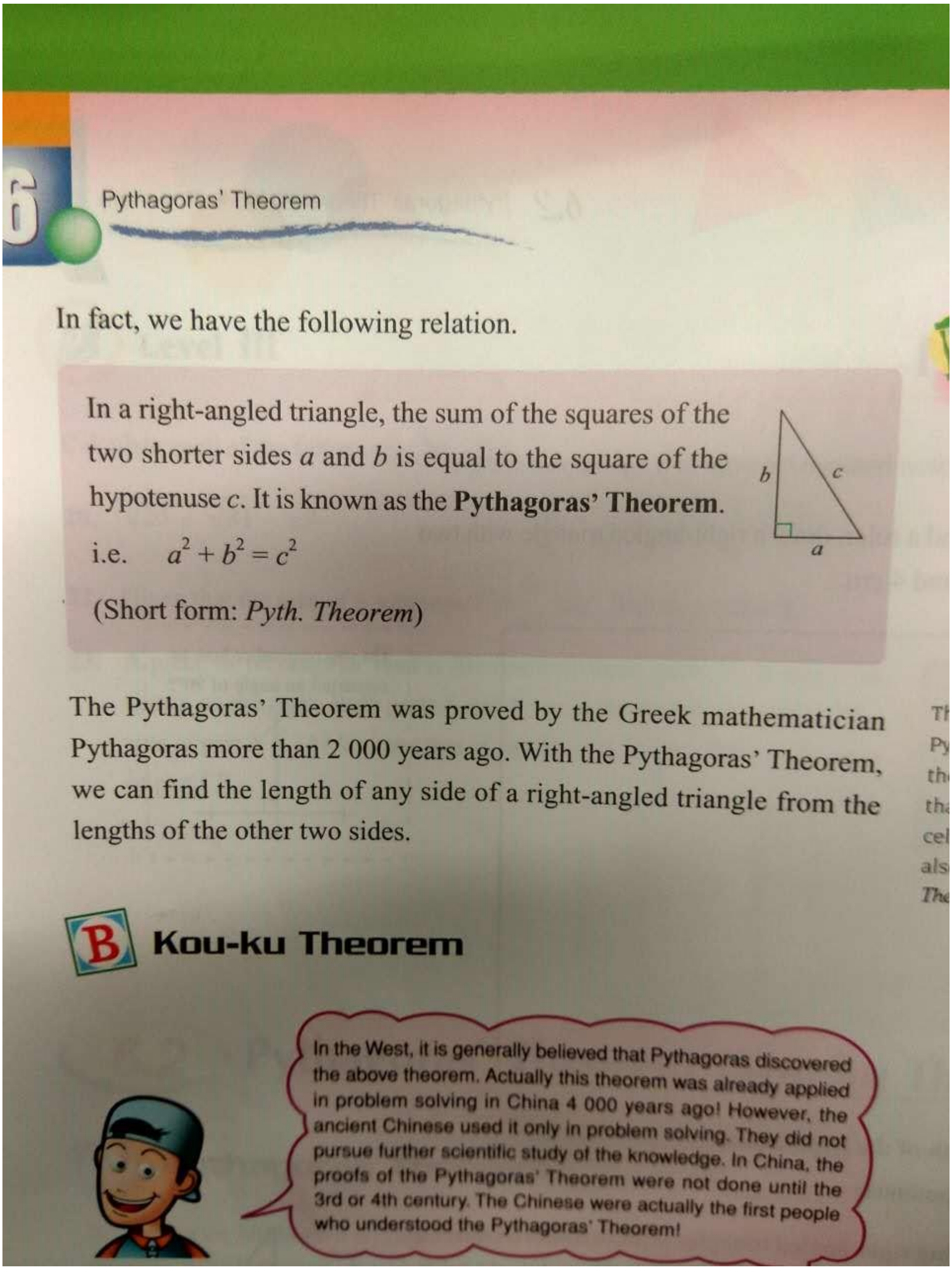
B. 74 cm

C. 150 cm

D. 164 cm



Appendix Four: Teaching of Pythagoras' Theorem in diverse mathematical textbooks



6 Pythagoras' Theorem

In fact, we have the following relation.

In a right-angled triangle, the sum of the squares of the two shorter sides a and b is equal to the square of the hypotenuse c. It is known as the **Pythagoras' Theorem**.

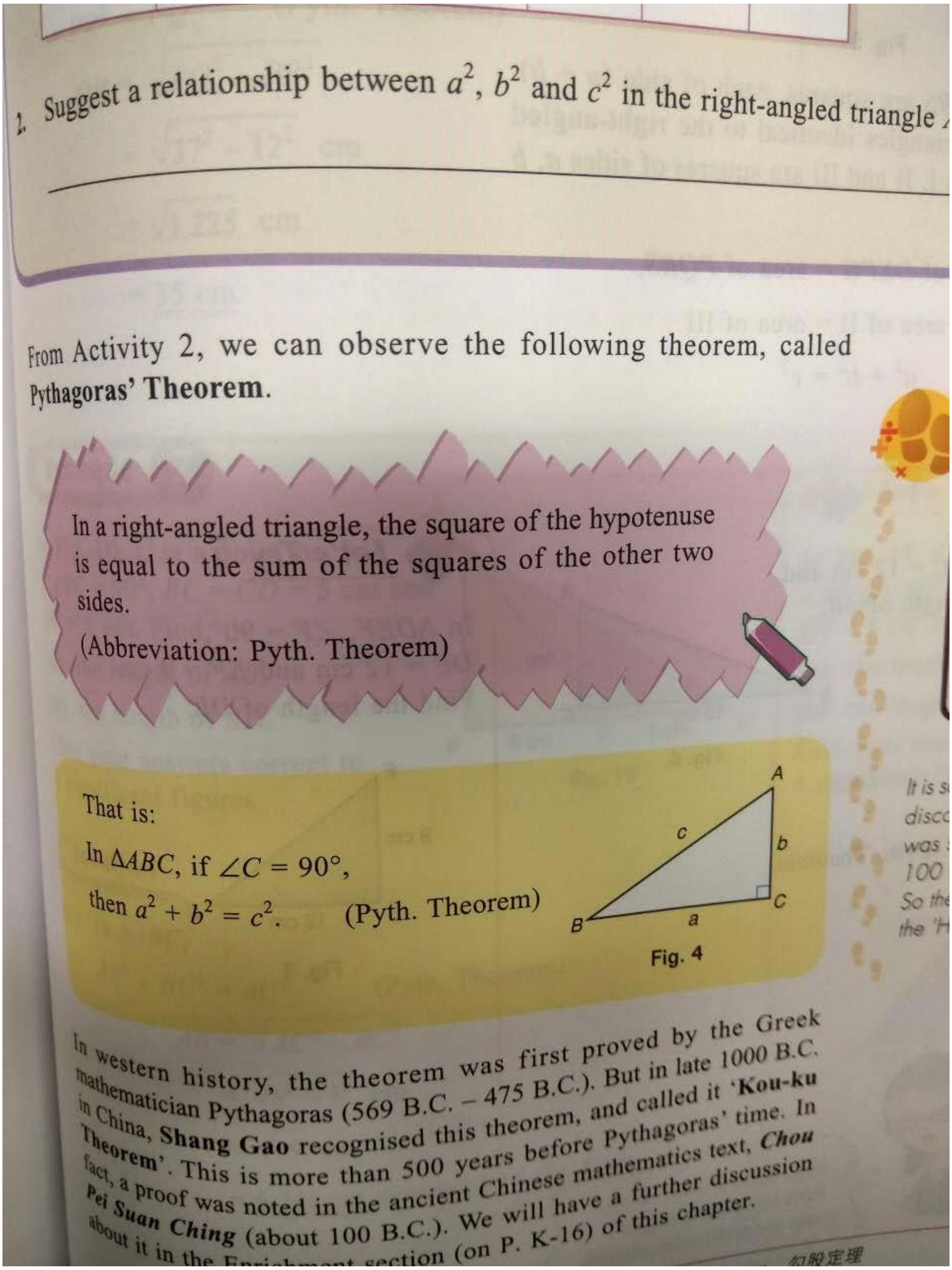
i.e. $a^2 + b^2 = c^2$

(Short form: *Pyth. Theorem*)

The Pythagoras' Theorem was proved by the Greek mathematician Pythagoras more than 2 000 years ago. With the Pythagoras' Theorem, we can find the length of any side of a right-angled triangle from the lengths of the other two sides.

B Kou-ku Theorem

Figure A4.1: Instance of teaching of Pythagoras' theorem in Chow (2005, p. 220)



2. Suggest a relationship between a^2, b^2 and c^2 in the right-angled triangle

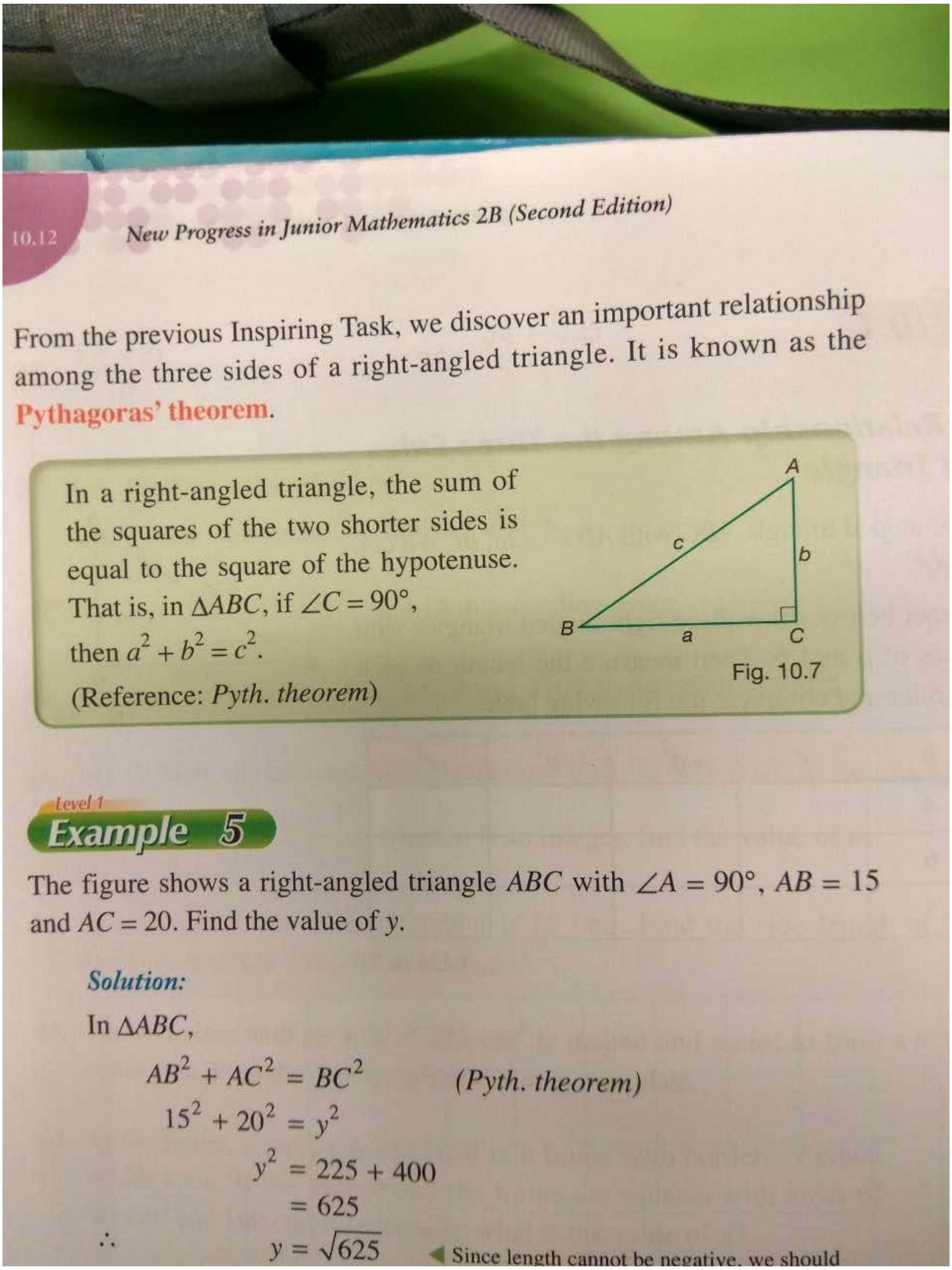
From Activity 2, we can observe the following theorem, called **Pythagoras' Theorem**.

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
(Abbreviation: Pyth. Theorem)

That is:
In ΔABC, if $\angle C = 90°$,
then $a^2 + b^2 = c^2$. (Pyth. Theorem)

In western history, the theorem was first proved by the Greek mathematician Pythagoras (569 B.C. – 475 B.C.). But in late 1000 B.C. in China, **Shang Gao** recognised this theorem, and called it '**Kou-ku Theorem**'. This is more than 500 years before Pythagoras' time. In fact, a proof was noted in the ancient Chinese mathematics text, ***Chou Pei Suan Ching*** (about 100 B.C.). We will have a further discussion about it in the Enrichment section (on P. K-16) of this chapter.

Figure A4.2: Instance of teaching of Pythagoras' theorem in Chow (2009, p. 11)



10.12 *New Progress in Junior Mathematics 2B (Second Edition)*

From the previous Inspiring Task, we discover an important relationship among the three sides of a right-angled triangle. It is known as the **Pythagoras' theorem**.

In a right-angled triangle, the sum of the squares of the two shorter sides is equal to the square of the hypotenuse. That is, in ΔABC, if $\angle C = 90°$, then $a^2 + b^2 = c^2$.
(Reference: *Pyth. theorem*)

Fig. 10.7

Level 1
Example 5

The figure shows a right-angled triangle ABC with $\angle A = 90°$, $AB = 15$ and $AC = 20$. Find the value of y.

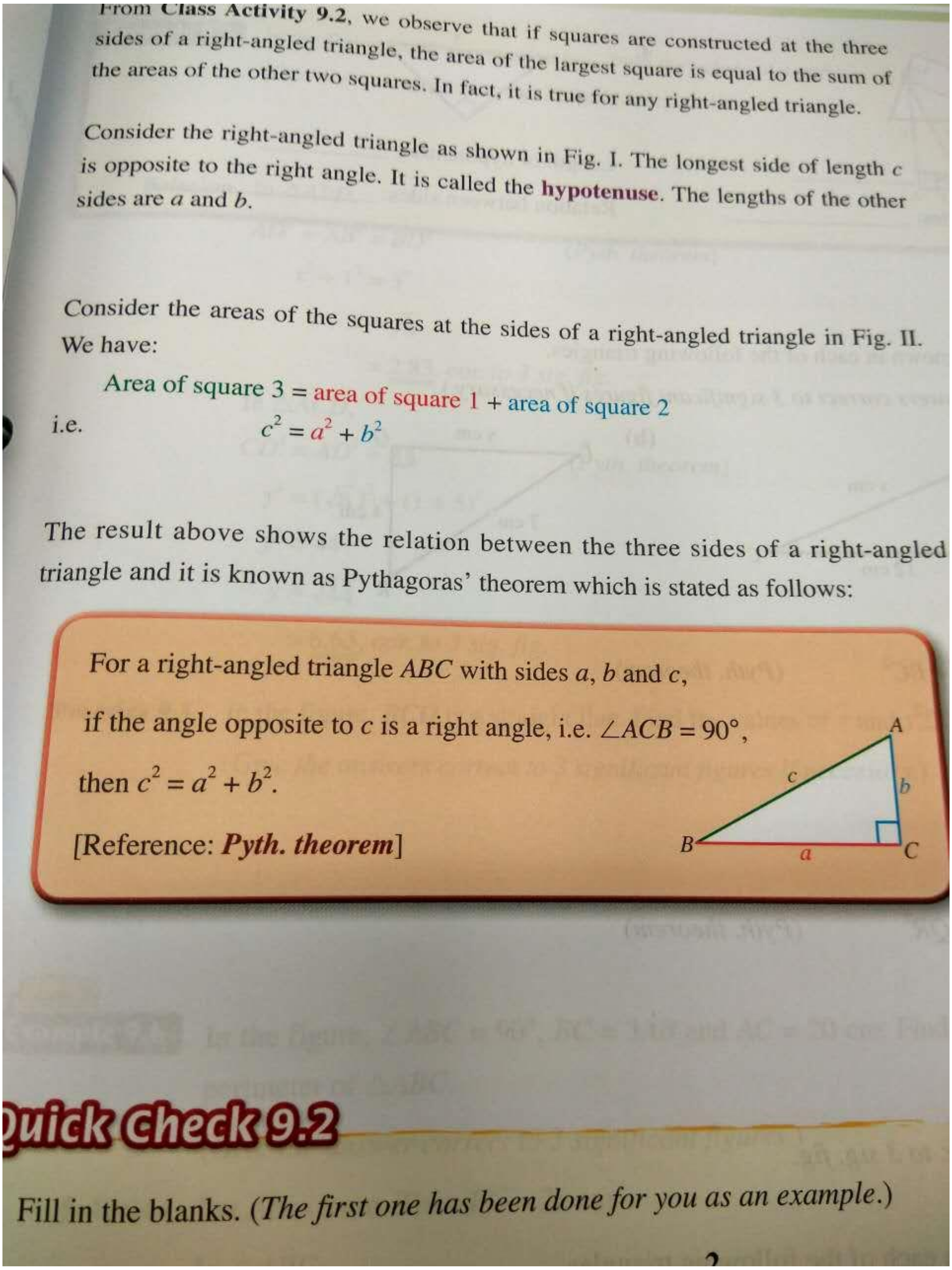
Solution:

In ΔABC,

$$AB^2 + AC^2 = BC^2 \quad \textit{(Pyth. theorem)}$$
$$15^2 + 20^2 = y^2$$
$$y^2 = 225 + 400$$
$$= 625$$
$$\therefore \quad y = \sqrt{625}$$

◀ Since length cannot be negative, we should

Figure A4.3: Instance of teaching of Pythagoras' theorem in Chan et al. (2008, p. 10)



From **Class Activity 9.2**, we observe that if squares are constructed at the three sides of a right-angled triangle, the area of the largest square is equal to the sum of the areas of the other two squares. In fact, it is true for any right-angled triangle.

Consider the right-angled triangle as shown in Fig. I. The longest side of length c is opposite to the right angle. It is called the **hypotenuse**. The lengths of the other sides are a and b.

Consider the areas of the squares at the sides of a right-angled triangle in Fig. II. We have:

Area of square 3 = area of square 1 + area of square 2

i.e. $c^2 = a^2 + b^2$

The result above shows the relation between the three sides of a right-angled triangle and it is known as Pythagoras' theorem which is stated as follows:

For a right-angled triangle ABC with sides a, b and c,

if the angle opposite to c is a right angle, i.e. $\angle ACB = 90°$,

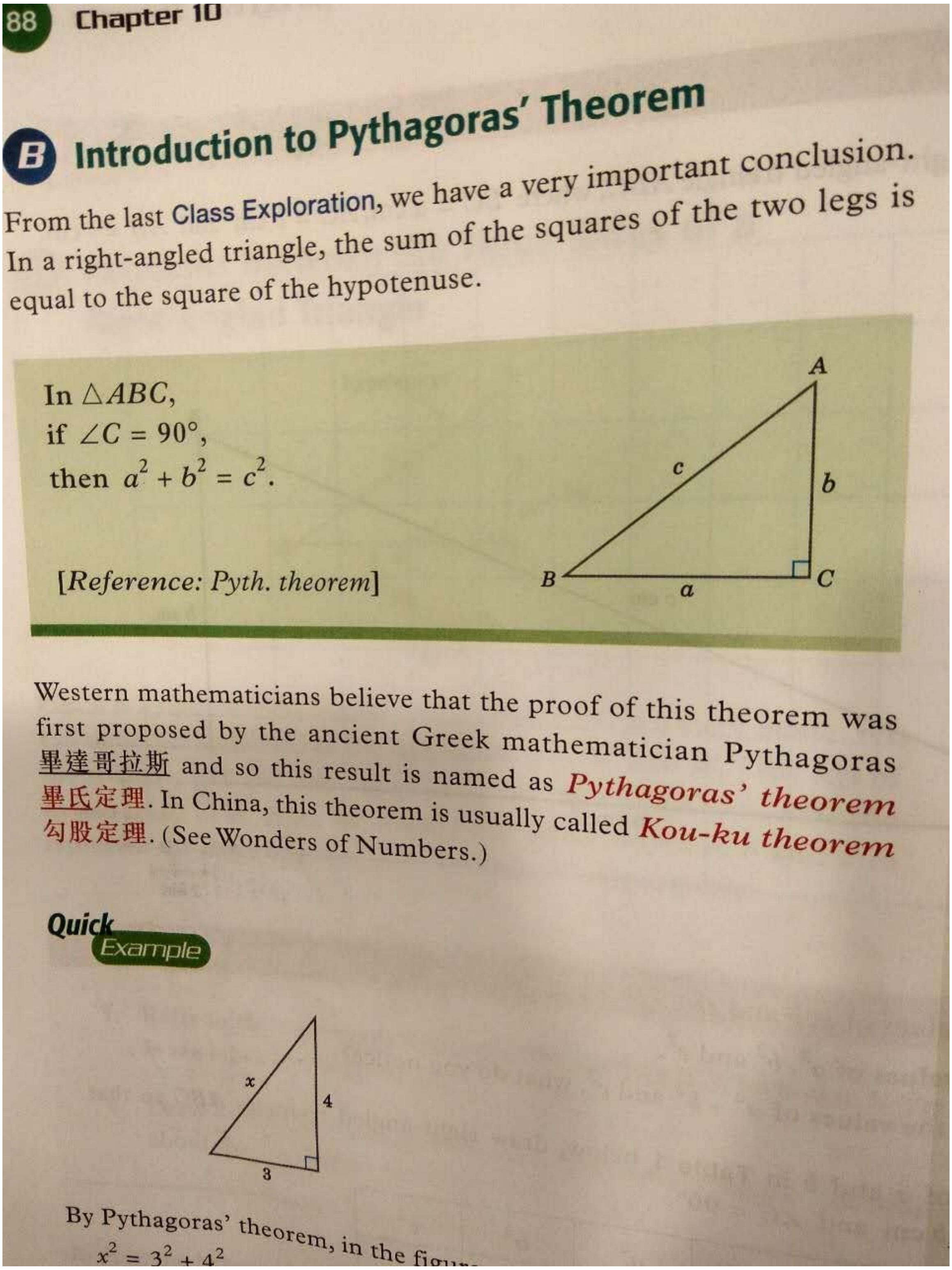
then $c^2 = a^2 + b^2$.

[Reference: ***Pyth. theorem***]

Quick Check 9.2

Fill in the blanks. (*The first one has been done for you as an example.*)

Figure A4.4: Instance of teaching of Pythagoras' theorem in Ho et al. (2013, p. 98)



88 Chapter 10

B Introduction to Pythagoras' Theorem

From the last Class Exploration, we have a very important conclusion. In a right-angled triangle, the sum of the squares of the two legs is equal to the square of the hypotenuse.

In $\triangle ABC$,
if $\angle C = 90°$,
then $a^2 + b^2 = c^2$.

[*Reference: Pyth. theorem*]

A, B, C, a, b, c

Western mathematicians believe that the proof of this theorem was first proposed by the ancient Greek mathematician Pythagoras 畢達哥拉斯 and so this result is named as ***Pythagoras' theorem*** 畢氏定理. In China, this theorem is usually called ***Kou-ku theorem*** 勾股定理. (See Wonders of Numbers.)

Quick Example

x, 4, 3

By Pythagoras' theorem, in the figu...

$x^2 = 3^2 + 4^2$

Figure A4.5: Instance of teaching of Pythagoras' theorem in Leung et al. (2005, p. 80)

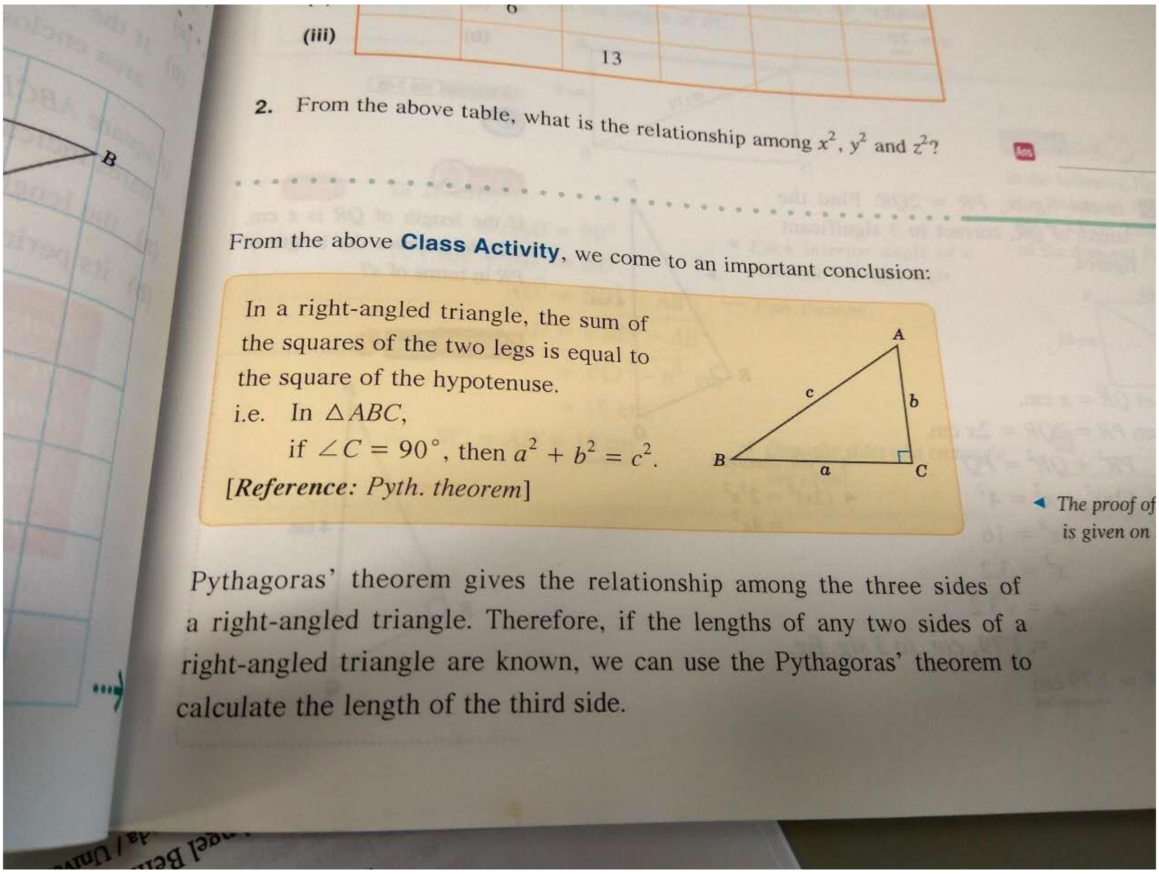
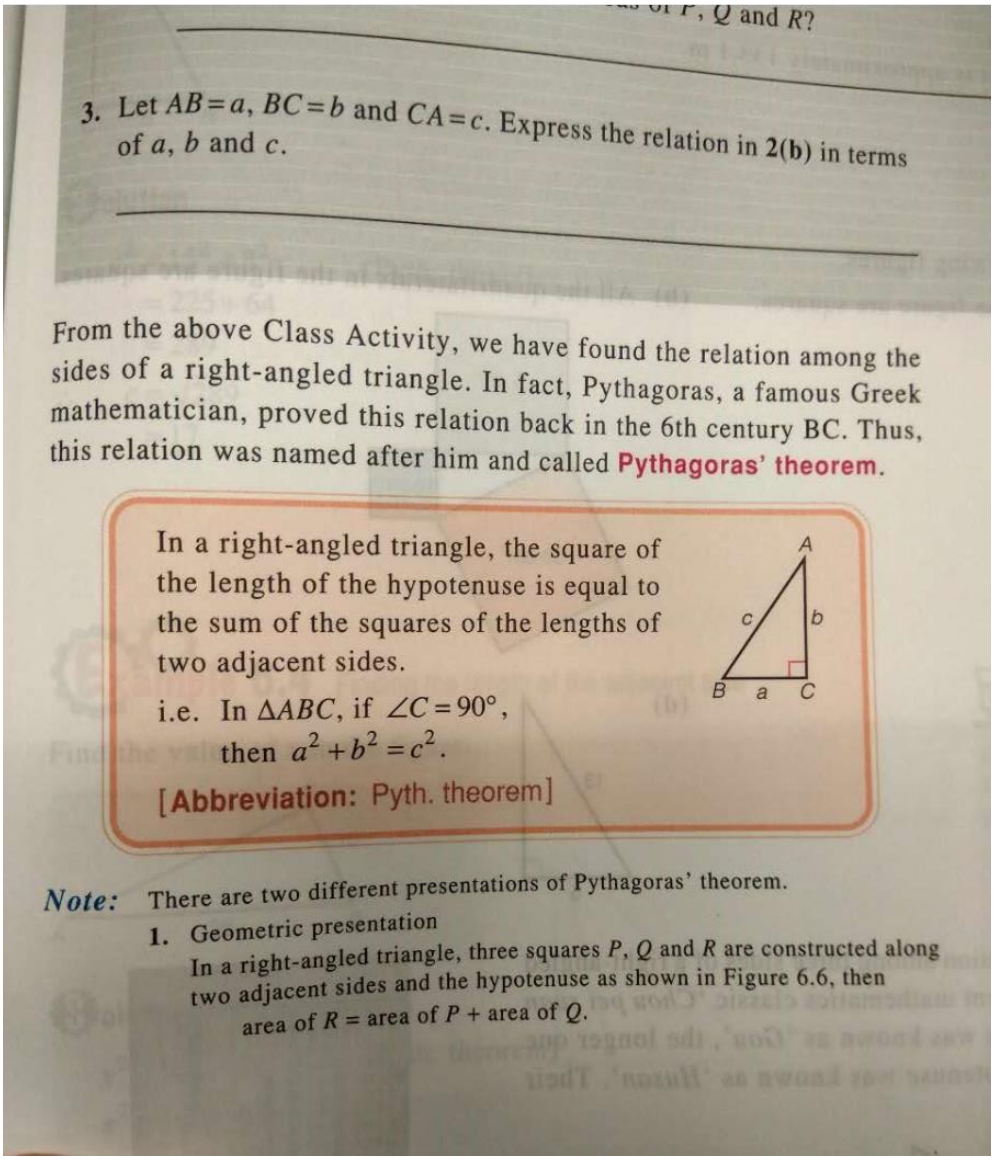


Figure A4.6: Instance of teaching of Pythagoras' theorem in Wong and Wong (2009), p. 103)



... of P, Q and R?

3. Let $AB = a$, $BC = b$ and $CA = c$. Express the relation in **2(b)** in terms of a, b and c.

From the above Class Activity, we have found the relation among the sides of a right-angled triangle. In fact, Pythagoras, a famous Greek mathematician, proved this relation back in the 6th century BC. Thus, this relation was named after him and called **Pythagoras' theorem**.

In a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of two adjacent sides.

i.e. In ΔABC, if $\angle C = 90°$,
then $a^2 + b^2 = c^2$.

[**Abbreviation:** Pyth. theorem]

Note: There are two different presentations of Pythagoras' theorem.

1. Geometric presentation
   In a right-angled triangle, three squares P, Q and R are constructed along two adjacent sides and the hypotenuse as shown in Figure 6.6, then
   area of R = area of P + area of Q.

Figure A4.7: Instance of teaching of Pythagoras' theorem in Chan et al. (2008), p. 69)

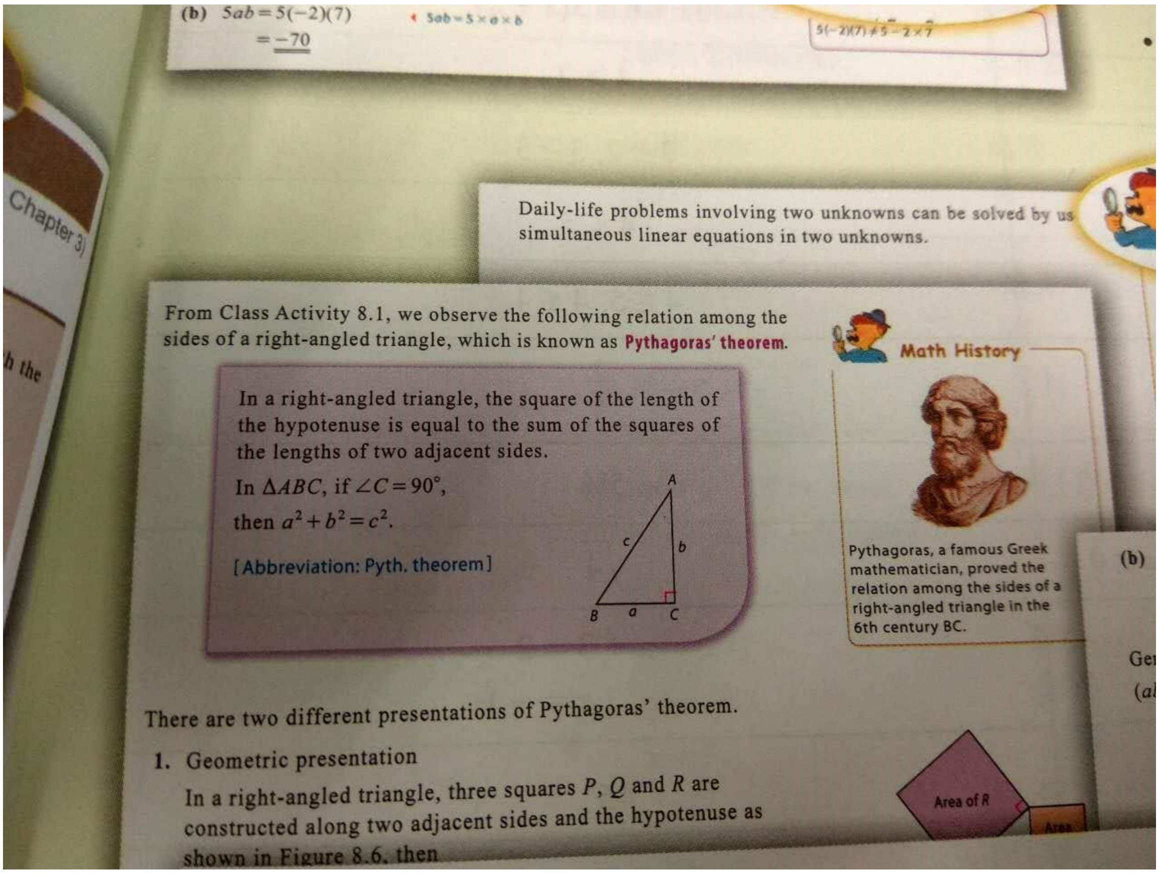
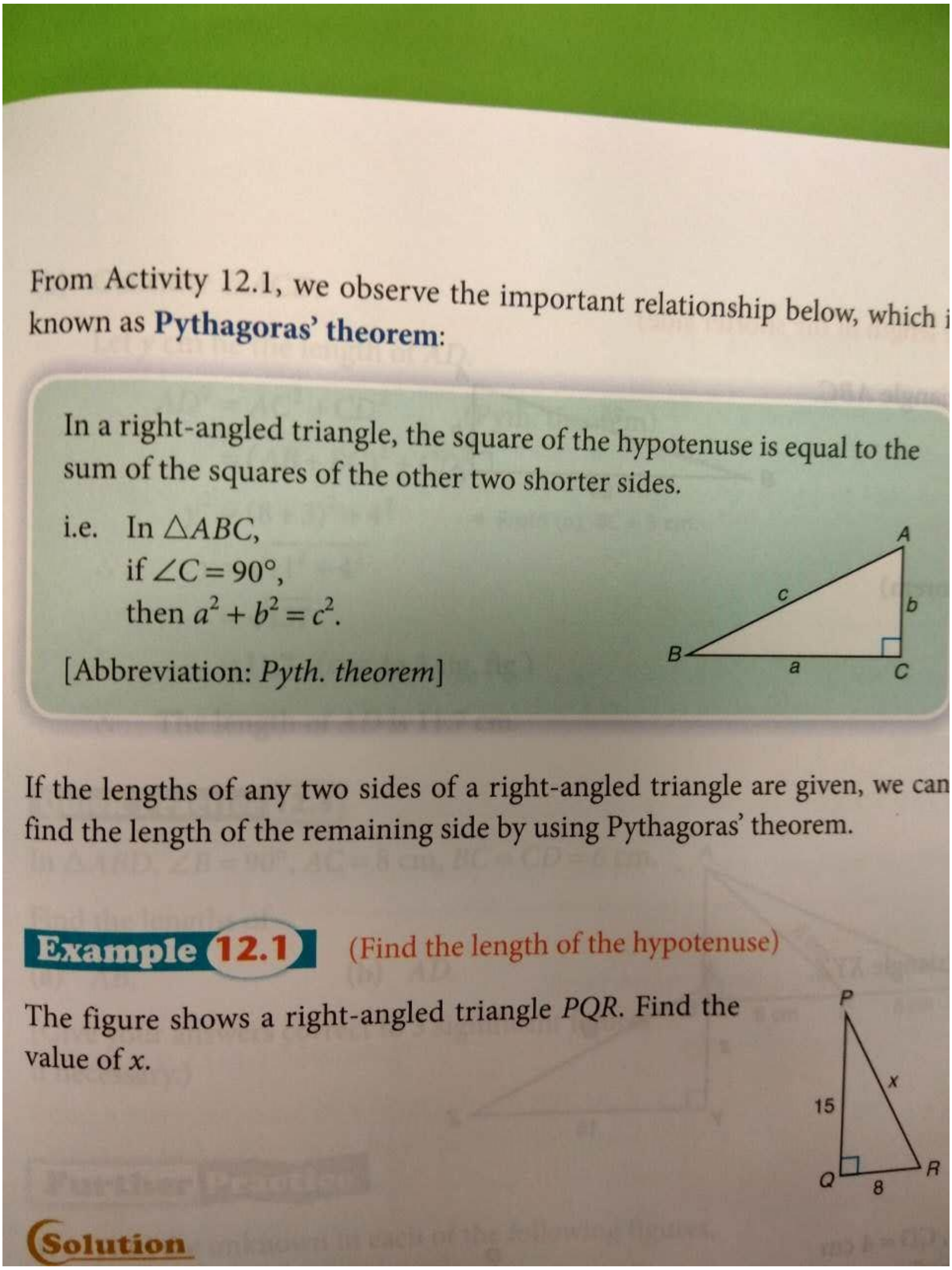


Figure A4.8: Instance of teaching of Pythagoras' theorem in Mui et al. (2015), p. xi)



From Activity 12.1, we observe the important relationship below, which i known as **Pythagoras' theorem**:

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two shorter sides.

i.e. In $\triangle ABC$,
if $\angle C = 90°$,
then $a^2 + b^2 = c^2$.

[Abbreviation: *Pyth. theorem*]

If the lengths of any two sides of a right-angled triangle are given, we can find the length of the remaining side by using Pythagoras' theorem.

Example 12.1 (Find the length of the hypotenuse)

The figure shows a right-angled triangle *PQR*. Find the value of *x*.

Solution

Figure A4.9: Instance of teaching of Pythagoras' theorem in Man et al. (2009, p. 369)